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ARGEM: a DSGE model with banks and monetary policy regimes with two feedback rules, calibrated for Argentina

Guillermo Escudé
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[1] The opinions expressed in this paper are the author's and do not necessarily reflect those of the institution to which he is affiliated.

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ARGEM: a DSGE model with Banks and Monetary Policy Regimes with two Feedback Rules, calibrated for Argentina[2]

1. Introduction

The last few years have seen an explosion of Dynamic and Stochastic General Equilibrium (DSGE) models built for policy analysis and forecasting in industrialized countries. The set of papers presented to the recent joint U.S. Federal Reserve Board-European Central Bank-IMF conference: "DSGE Modeling at Policymaking Institutions: Progress & Prospects" is a significant sample. The need for better microfounded models that can contribute to policy analysis is also experienced by developing country Central Banks, Argentina being no exception. On top of the many difficulties encountered in developed countries in building, calibrating and/or estimating these models, those who seek to construct models that can be relevant in the developing country context find various additional difficulties. One of these stems from the fact that the models built for industrialized countries typically assume a freely floating exchange rate and hence can avoid modeling exchange rate policy. Most developing countries do not have a pure exchange rate float and their Central Banks regularly intervene in the foreign exchange market with varying degrees of intensity and frequency. While the opposite "corner" of a pure interest rate float with a monetary policy based on determining a path for the nominal exchange rate is not difficult to model, one of the challenges faced by developing country modelers is to incorporate intervention in the foreign exchange market as an additional tool available for a Central Bank that also intervenes in the "money" market (typically by determining an operational target for the short run interest rate). This is one of the main objectives of this paper, which on this topic builds on previous analysis by the author (see Escudé (2006)). The paper benefits from various recent developments in monetary macroeconomic modeling, including Christiano, Eichenbaum and Evans (2001) (CEE), Smets and Wouters (2003), Woodford (2003), and Adolfson, Laséen, Lindé and Villani (2005) (ALLV), to mention but a few.

As is typical in recent DSGE models, ARGEM has various nominal and real rigidities that help to achieve realistic dynamics: habit formation in consumption, adjustment costs in investment, costs for abnormal intensity in the utilization of physical capital, transactions costs, risk premia by foreign lenders, Calvo-Yun-Rotemberg wage and price setting with full indexation to the previous period's inflation for non-optimizers, gradual path-through of import costs (including the exchange rate) to domestic prices as well as gradual path-through of domestic costs to foreign currency pricing for exporters of manufactures. Some of these rigidities generate a role for (nominal or real) exchange rate stabilization. The empirical evidence on incomplete short run exchange rate pass-through led Smets and Wouters (2002), for example, to explore its implications for optimal monetary policy in the open economy through a DSGE model calibrated to the euro area. They show that minimizing the welfare costs that arise due to gradual pass-through introduces a justification for including exchange rate stabilization in the Central Bank's loss function. The welfare cost of exchange rate variability thus operates in

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the opposite direction to the need for exchange rate flexibility in order to overcome the ineffectiveness of the exchange rate channel as a result of a gradual pass-through. Such evidence has led many researchers to include an exchange rate response in simple monetary policy feedback rules. This is the case of the estimated models of ALLV (2005) for the euro area and, in the developing country context, of Caputo, Liendo and Medina (2007) for Chile in the inflation targeting period.

To the traditional Central Bank interest rate instrument that responds to exchange rate developments, we add a more direct foreign exchange intervention through the sale and purchase of international reserves. This has various possible justifications. On the one hand, it is an empirical fact that this instrument is used by many developing country central banks (and also many central banks in industrialized economies (see Bofinger and Wollmershaüser (2001)). On the other hand, it seems intuitively plausible that two instruments should allow the central bank to better achieve its objectives, for example, obtaining a lower loss for a given intertemporal quadratic loss function. In the model we present, the interest rate instrument impacts directly on the banking system since the central bank's interest rate instrument is the rate that defines banks' deposit and lending margins and hence rates. While the deposit rate affects households' saving/expenditure decision as well as the amount of cash they wish to hold (since they save in bank deposits and the deposit interest rate is their opportunity cost for holding cash), the lending rate directly affects domestic sector firms' marginal costs, since these firms finance a part of their variable costs through bank loans. The inclusion of a banking sector also enriches the monetary policy transmission mechanism through other channels. In particular, it allows for the introduction of a regulatory prudential requirement that directly affects banks' deposit margin. And since banks also invest in central bank bonds, it allows for a consistent modelling of foreign exchange market intervention sterilization. Furthermore, the role of the banking system is enhanced by the fact that the model's uncovered interest parity condition derives from banks' profit maximization and their obtaining funding abroad under a risk premium. However, the central bank's exchange market intervention also affects the real sector by directly smoothing fluctuations in the real exchange rate that impact on households' consumption and investment decisions. This smoothing complements the smoothing that takes place owing to import firms' incomplete pass-through to import prices. Indeed, the central bank's foreign exchange intervention has the potential to modify the smoothing that such pricing practices of importing firms achieves in order to better attain its objectives, whatever they may be. The two separate instruments hence impact the economy through basically different mechanisms (that are of course interrelated) and have their direct impact on different places: the interest rate instrument impacts directly on the banking system, and the foreign exchange market intervention impacts more directly on the export and import sectors by affecting the banks' arbitrage activity.

The main features of the model are the following: 1) The Central Bank exercises an Inflation Targeting with a Managed Exchange rate Float (IT_MEF) regime that, in the "corners" includes a crawling peg policy and inflation targeting with a pure exchange rate float. By IT-MEF we mean that even though the inflation target is the nominal anchor, the Central Bank simultaneously intervenes in the foreign exchange and money markets with two parallel feedback policy rules. 2)

Growth is driven by a permanent productivity shock. In the theoretical model we assume that there is cointegration between the (logs of the) small domestic economy's (SOE) unit root technology shock and the large rest of the world's (LRW). However, in a simpler version without cointegration we assume that the relative productivity shock between the SOE and the LRW is an exogenous autorregressive process, as in ALLV (2005). 3) Households do not engage in external debt nor save in foreign assets. The financial closure of the SOE is instead based on the government's and banks' use of foreign funding, the cost of which is increasing in their (detrended) level of net debt. A risk-adjusted uncovered interest parity condition naturally stems from banks' profit maximization. 4) There is a full fledged banking system. Banks' cost is a function of their loan and deposit stocks, with economies of scope between lending and deposit taking activities. These economies of scope were introduced in order to allow for a realistic calibration of the bank parameters. Banks have a technical demand for cash, which is a (possibly stochastic and time-varying) fraction of deposits, and must keep a regulatory fraction of their deposits in non-interest bearing reserves in the Central Bank. They use the remaining fraction of their deposits as well as foreign funds to finance firms' demand for loans and the Government's exogenous demand for loans, to purchase Central Bank bonds, and to lend (or borrow) in the interbank market. To build inertia into the "uncovered interest parity condition" we assume that a fraction of the banks, instead of forming expectations rationally, have static expectations with respect to nominal currency depreciation. 5) The tax structure is minimal (just lump sum taxes), but the government can also finance its expenditures by issuing debt abroad, by obtaining bank loans, and by using the Central Bank's quasi-fiscal surplus. Fiscal policy is assumed to be coherent enough to allow for a non-fiscally dominated monetary policy. 6) In order to capture the effects of changes in commodity prices, we include a sector of primary goods producers (see Murchison and Rennison (2005)). These firms are price takers and produce under diminishing returns due to their fixed endowments of natural resources, using physical capital services and domestic goods as inputs. They sell their output both domestically as inputs for domestic firms and abroad. 7) Firms in the domestic sector are monopolistic competitors. The production of intermediate domestic goods requires the use of produced primary goods and imported goods as inputs in addition to labor and physical capital services. These firms obtain bank loans a period in advance to finance time-varying stochastic fractions of their expected expenditures on rents, wages, primary inputs, and imported inputs. 8) Households own the physical capital stock, which is generated through a technology that converts investment into physical capital. They rent the physical capital to firms for a rental price that is determined in a competitive market, and also determine the intensity at which firms are to use it (see CEE (2001)). Households save through bank deposits. They use cash for consumption and investment spending using a stylized transactions technology that requires the use of domestic goods. Hence, cash is not in the utility function, and the resulting household demand for cash is dependent on private absorption and the deposit interest rate. 9) There are two kinds of goods exported: primary and manufactured. Primary sector firms export all output that is not sold to the domestic sector at the (exogenous) international price. The Law of One Price holds for these firms. On the other hand, firms that export manufactured goods differentiate the domestic goods bundle (which is their input) and sell in the LRW through sticky local currency pricing (see ALLV (2005)). Importing firms differentiate the bundle of goods produced abroad to sell in the SOE through sticky local currency pricing. 10) We use the Calvo (1983) framework to distinguish firms (households) that can set prices (wages) by optimization, and full indexation to lagged inflation (see CEE (2001)) for those that can't optimize currently.

The model has been calibrated for the Argentine economy[3], taking the year as the unit time period, and solved using Klein's (2000) generalized Schur decomposition methodology.

The rest of this paper has the following structure. Section 2 presents the household optimization problem, which determines their consumption and investment demands, the rate of utilization of physical capital, the dynamics of the stock of physical capital, their cash and bank deposit demands, and their nominal wage setting. The latter generates a Phillips equation for wage inflation. Section 3 presents the decisions of domestic goods producers, including their demand for labor and physical capital services, their demand for imported inputs and bank funding, their supply of goods and their nominal price setting. The latter generates a Phillips equation for domestic inflation. Section 4 has the decisions of primary goods producing firms. Section 5 has the decisions of foreign trade firms, which generate respective Phillips equations for manufactured export goods and for imported goods. Section 6 summarizes the main relative prices that pertain to the SOE's relation with the LRW. Section 7 models banks' decision problem, which determines their demand for cash and required reserves, their demand for foreign funds and for Central Bank bonds, and their supply of deposits and loans. Section 8 introduces the public sector, composed of the Government and the Central Bank. The Central Bank balance sheet plays a significant role in the modeling of the simultaneous intervention in the money and foreign exchange markets. Section 9 puts together the market clearing equations, the balance of payments equation, and the relation between the domestic sector output and GDP. Section 10 addresses the Central Banks' monetary/foreign exchange policy. Section 11 lists the non-policy equations of the non-linear system so far encountered. Section 12 transforms this set of equations so that the variables are in stationary format, and adds the policy equations. Section 13 performs an analysis of the non-stochastic steady state around which we make the log-linear approximation. Section 14 states our assumptions on the stochastic shocks that impinge on the economy, with emphasis on those pertaining to the (exogenous) growth producing technological progress. Section 15 presents the functional forms for the various auxiliary functions used in the calibrated model, namely, the investment adjustment cost function, the function that reflects the costs due to non-normal intensity of utilization of physical capital, the transactions cost function, and Banks' and the Government's risk premium function. Section 16 presents the complete log-linearized system and puts it in a matrix form suitable for the numerical solution using the generalized Schur

[3] The detailed calibration process is to be included in a forthcoming paper. The calibration of the persistence parameters of the exogenous autorregressive shocks used in this paper is preliminary. The calibrated values of the parameters used in this paper are shown in Appendix 2.

decomposition. Finally, section 17 concludes. The paper has four Appendixes. The first contains the details of the more cumbersome log-linearizations: the Phillips equations for domestic goods inflation and wage inflation. The second lists the parameters and great ratios used in the calibration as well as their numerical values. The third lists the definitions and resulting numerical values of the equation coefficients that result from the log-linearization of the model equations. Finally, the fourth contains the graphs of the impulse response functions.

2. Households

Infinitely lived households are monopolistic competitors in the supply of differentiated labor. There is a domestic market for state-contingent securities that are held by households, insuring them against profit and wage idiosyncratic risks (see Woodford (2003)). This makes households essentially the same in equilibrium, and allows us to maintain the representative household fiction (i.e. dispense with the complexities that stem from household heterogeneity). Aside from these state-contingent securities, they hold financial net wealth in the form of domestic currency ($M_t^{0,H}$), and peso denominated one period nominal deposits issued by domestic commercial banks (D_t) that pay a nominal interest rate i_t^D. We assume that the Central Bank fully and credibly insures depositors, so the deposit rate is considered riskless. Households also invest a real amount V_t to expand the stock of capital goods that they own and rent to firms, earning each period a real rental price i_t^K.

2.1. Physical capital, investment, and the rate of capital utilization

Each household h decides at t the rate of gross investment $V_t(h)$, which contributes to the determination of the quantity of physical capital K_{t+1} in period $t+1$ through the following law of motion for the stock of physical capital:

$$K_{t+1}(h) = \left(1 - \delta^K\right) K_t(h) + z_t^V V_t(h) \left[1 - \tau_V \left(\frac{V_t(h)}{V_{t-1}(h)}\right)\right], \tag{1}$$

where δ^K is the (constant) rate of capital depreciation, and z_t^V is an economy wide stationary investment efficiency shock. As in CEE (2001), the second term on the right hand side is a representation of the technology that transforms investment goods into capital goods. These capital goods are rented by households to firms. We have no market for capital goods in the model and hence no explicit price for these goods. As we see below, we do have a shadow price for installed physical capital (as well as a rental rate). The function $\tau_V(.)$ represents convex investment adjustment costs for off-steady state situations. Technically, we assume that when evaluated in the steady state rate of growth of V_t (which is μ^{z**}), both τ_V and its derivative are zero:

$$\tau_V\left(\mu^{z**}\right) = \tau_V'\left(\mu^{z**}\right) = 0, \qquad \tau_V''\left(\mu^{z**}\right) > 0. \tag{2}$$

The household decision process includes establishing the rate of capital utilization intensity that firms will use (and pay for) in period t for the stock of physical capital it rents. As CEE (2001) argue, allowing for elastic capital utilization has the beneficial properties of 1) dampening movements in marginal cost by reducing fluctuations in the rental rate of physical capital i_t^K and also 2) reducing the fluctuations in labor productivity after monetary policy shocks (see also Smets and Wouters (2002)). Let u_t represent the rate of capital utilization. Hence, the flow of physical capital services that firms use as input is:

$$u_t K_t \equiv K_t^u.$$

Using a rate of utilization of capital that exceeds the normal (steady state) level, however, is costly (whereas a lower than normal utilization actually implies a savings in total cost) and impinges in the net return from renting. Let $\tau_u(u_t)$ be the amount of real resources (domestic goods) used up when the rate of utilization is u_t. We assume that this function is increasing and convex, and normalize units so that the steady state rate of utilization is unity, at which there are no costs (or savings):

$$\tau_u'(u_t) > 0, \tau_u''(u_t) > 0 \text{ and } \tau_u(1) = 0. \tag{3}$$

Hence, taking abnormal utilization costs into account, the net return from renting $K_t(h)$ units of capital is:

$$\left[i_t^K u_t(h) - \tau_u(u_t(h))\right] K_t(h). \tag{4}$$

2.2. Transaction costs

The household holds cash $M_t^{0,H}$ because doing so it economizes on transaction costs. We assume that consumption and investment related transactions involve the use of real resources (domestic goods) and that these transaction costs per unit of expenditure in consumption and investment goods (private absorption) are a decreasing and convex function τ_M of the currency/absorption ratio ϖ_t (see Feenstra (1986)):

$$\tau_M(\varpi_t) \qquad\qquad \tau_M' < 0, \tau_M'' > 0,$$

$$\varpi_t \equiv \frac{M_t^{0,H}(h)}{P_t^C C_t(h) + P_t^V V_t(h)} = \frac{M_t^{0,H}(h)/P_t}{p_t^C C_t(h) + p_t^V V_t(h)}$$

where C_t is consumption (of private goods), and P_t, P_t^C and P_t^V are the price indexes of domestic, consumption, and investment goods, respectively. All price indexes are in monetary units. The two basic price indexes in the SOE are those of domestically produced ('domestic') goods, P_t, and imported goods P_t^N. The consumption and investment price indexes are both CES composites of these basic price indexes, as we elaborate below. For convenience, we define the relative prices of consumption and investment goods in terms of domestic goods:

$$p_t^C \equiv \frac{P_t^C}{P_t}, \qquad p_t^V \equiv \frac{P_t^V}{P_t}.$$

When the currency/absorption ratio increases, transaction costs per unit of absorption decrease at a decreasing rate, reflecting a diminishing marginal productivity of currency in reducing transaction costs.

2.3. Sticky nominal wage setting

We model nominal stickiness as in Calvo (1983), adapted to discrete time (Rotemberg (1987)) and extended to (full) indexation (Yun (1996) and Christiano, Eichenbaum and Evans (2001)). Household $h \in [0,1]$ supplies labor of type h, and makes the wage setting decision taking the aggregate wage index and labor supply as parametric. Every period, each household has a probability $1-\alpha_W$ of being able to set the optimum wage for its specific labor type. This probability is independent of when it last set the optimal wage. When it can't optimize, the household adjusts its wage rate by fully indexing to last period's overall rate of wage inflation. Hence, when it can set the optimal wage rate it must take into account that in any future period j there is a probability α_W^j that its wage will be the one it sets today plus full indexation. Hence, the household faces a wage survival constraint, according to which the wage rate it sets at t, $W_t(h)$, has a probability α_W^j of surviving (indexed) until period $t+j$:

$$\begin{aligned} W_{t+j}(h) &= W_t(h)\frac{W_t}{W_{t-1}}\frac{W_{t+1}}{W_t}...\frac{W_{t+j-1}}{W_{t+j-2}} \\ &\equiv W_t(h)\pi_t^w\pi_{t+1}^w...\pi_{t+j-1}^w \equiv W_t(h)\Psi_{t,j}^w, \end{aligned} \tag{5}$$

where we define the rate of wage inflation $\pi_t^w \equiv W_t/W_{t-1}$, and the cumulative wage inflation between $t+j-2$ and t, $\Psi_{t,j}^w$, with $\Psi_{t,0}^w \equiv 1$. In deriving the first order condition for $W_t(h)$ below we use the following identity:

$$\frac{W_t(h)}{W_{t+j}}\Psi_{t,j}^w = \frac{W_t(h)}{W_t}\frac{\pi_t^w\pi_{t+1}^w...\pi_{t+j-1}^w}{\pi_{t+j}^w\pi_{t+j-1}^w...\pi_{t+1}^w} = \frac{W_t(h)}{W_t}\frac{\pi_t^w}{\pi_{t+j}^w}. \tag{6}$$

Another constraint the household faces is its labor demand function:

$$h_t(h) = h_t\left(\frac{W_t(h)}{W_t}\right)^{-\psi}, \tag{7}$$

where W_t is the aggregate wage index, defined as:

$$W_t = \left\{\int_0^\infty W_t(h)^{1-\psi}dh\right\}^{1/(1-\psi)}, \tag{8}$$

and where ψ is the elasticity of substitution between differentiated labor services[4]. When h sets the optimal wage, it must take into account that there is a probability α_W^j that at time $t+j$ its wage will be the $W_t(h)\Psi_{t,j}^w$, and that hence the labor demand it faces is:

$$h_{t+j}(h) = h_{t+j}\left(\frac{W_t(h)\Psi_{t,j}^w}{W_{t+j}}\right)^{-\psi}. \tag{9}$$

2.4. The household optimization problem

The household receives income from profits, wage, rent, and interest, and spends on consumption, investment, taxes, and transaction costs. It's real budget constraint

[4] We derive these equations from domestic intermediate firms' cost minimization in section 3.2 below.

in period t is:

$$\frac{M_t^{0,H}(h)}{P_t}+\frac{D_t(h)}{P_t}=\frac{\Pi_t(h)}{P_t}+\frac{W_t(h)}{P_t}h_t(h)-\frac{T_t(h)}{P_t}+\frac{\Upsilon_t(h)}{P_t} \tag{10}$$
$$+\left[i^K u_t(h)-\tau_u(u_t(h))\right]K_t(h)+\frac{M_{t-1}^{0,H}(h)}{P_t}+\left(1+i_{t-1}^D\right)\frac{D_{t-1}(h)}{P_t}$$
$$-\left[1+\tau_M\left(\frac{M_t^{0,H}(h)/P_t}{p_t^C C_t(h)+p_t^V V_t(h)}\right)\right]\left(p_t^C C_t(h)+p_t^V V_t(h)\right)$$

where $\Pi_t(h)$ is nominal profits, $h_t(h)$ is hours of labor exertion, $T_t(h)$ is lump sum taxes net of transfers, and $\Upsilon_t(h)$ is the income obtained in t from holding state-contingent securities. Our timing convention concerning cash and deposits differs from CEE (2001). In our case, the household at t chooses its time t (instead of time t+1) cash and deposit demands. This has the advantage of making our banking model (which is considerably more complicated than in CEE (2001)) consistent at the cost of turning dynamic some of the equations in the domestic firm and bank sectors that would otherwise be static.

Household h maximizes an inter-temporal utility function which is additively separable in the consumption of private goods C_t, public goods C_t^G, and leisure:

$$E_t\sum_{j=0}^{\infty}\beta^j\{z_{t+j}^C\log\left[C_{t+j}(h)-\xi C_{t+j-1}(h)\right]+ \tag{11}$$
$$+\eta_G\log\left[C_{t+j}^G(h)-\xi_G C_{t+j-1}^G(h)\right]+[\bar{h}-\frac{\eta_H z_{t+j}^H}{1+\chi}h_{t+j}(h)^{1+\chi}]\},$$

where β is the intertemporal discount factor, $\bar{h}$ is the maximum labor time available (and hence the last term in square brackets is "leisure"), and z_t^C and z_t^H are consumption demand and labor supply shocks that are common to all households. Consumption nests habit formation, where ξ and ξ_G are less than unity (see Fuhrer (2000) and Christiano, Eichenbaum and Evans (2001)) into a log utility function. Consumers hence care about both their level of consumption and their rate of consumption growth. Since the consumption of public goods is not a decision variable for the household, the term that includes it is only relevant for the evaluation of the welfare effects of alternative fiscal policies. We drop it below for simplicity.

The household's inter-temporal solvency is guaranteed by its inability to incur in debt, which we assume does not bind in any finite time:

$$D_{t+T}\geq 0,\qquad \forall T\geq 0. \tag{12}$$

Household h chooses $C_{t+j}(h)$, $V_{t+j}(h)$, $K_{t+1+j}(h)$, $u_{t+j}(h)$, $D_{t+j}(h)$, $M_{t+j}^{0,H}(h)$, (j=1,2,...) and $W_t(h)$, to maximize (11) subject to its sequence of budget constraints (10), physical capital accumulation constraints (1), its combined labor demands and wage survival constraints (9), and its "no debt" constraints (12).

The Lagrangian is hence:

$$
\begin{aligned}
& E_t \sum_{j=0}^{\infty} (\beta \alpha_W)^j \{ z^C_{t+j} \log \left[C_{t+j}(h) - \xi C_{t+j-1}(h) \right] + \overline{h} \\
& - \frac{\eta_H z^H_{t+j}}{1+\chi} \left(h_{t+j} \left(\frac{W_t(h)\Psi^w_{t,j}}{W_{t+j}} \right)^{-\psi} \right)^{1+\chi} + \lambda_{t+j}(h) \{ \frac{\Pi_{t+j}(h)}{P_{t+j}} - \frac{T_{t+j}(h)}{P_{t+j}} \\
& + \frac{W_t(h)\Psi^w_{t,j}}{P_{t+j}} h_{t+j} \left(\frac{W_t(h)\Psi^w_{t,j}}{W_{t+j}} \right)^{-\psi} + \left[i^K_{t+j} u_{t+j}(h) - \tau_u(u_{t+j}(h)) \right] K_{t+j}(h) \\
& - \left[1 + \tau_M \left(\frac{M^{0,H}_{t+j}(h)/P_{t+j}}{p^C_{t+j} C_{t+j}(h) + p^V_{t+j} V_{t+j}(h)} \right) \right] \left(p^C_{t+j} C_{t+j}(h) + p^V_{t+j} V_{t+j}(h) \right) \\
& + \frac{M^{0,H}_{t+j-1}(h)}{P_{t+j}} + \left(1 + i^D_{t+j-1} \right) \frac{D_{t+j-1}(h)}{P_{t+j}} - \frac{M^{0,H}_{t+j}(h)}{P_{t+j}} - \frac{D_{t+j}(h)}{P_{t+j}} + \frac{\Upsilon_{t+j}(h)}{P_{t+j}} \} \\
& + \zeta_{t+j}(h) \{ \left(1 - \delta^K \right) K_{t+j}(h) + z^V_{t+j} V_{t+j}(h) \left[1 - \tau_V \left(\frac{V_{t+j}(h)}{V_{t-1+j}(h)} \right) \right] \\
& - K_{t+1+j}(h) \} \} .
\end{aligned}
\tag{13}
$$

where $\beta^j \lambda_{t+j}(h)$ and $\beta^j \zeta_{t+j}(h)$ are the Lagrange multipliers (for the budget constraints and the capital accumulation constraints), which can be interpreted as the marginal utility of real income, and the shadow price of installed physical capital, respectively. We will refer to λ_t and ζ_t as the undiscounted Lagrange multipliers.

2.5. First order conditions

Since households only differ on whether they can choose the optimal wage, we eliminate the household index, and use $\widetilde{W}_t$ to distinguish the newly optimal wage from the aggregate wage index W_t (which includes both optimal and indexed wages). The first order conditions for an optimum (including the transversality condition) are the following:

$$
C_t: \quad \frac{z^C_t}{C_t - \xi C_{t-1}} - \beta \xi E_t \left(\frac{z^C_{t+1}}{C_{t+1} - \xi C_t} \right) = \lambda_t \varphi_M \left(\frac{M^{0,H}_t / P_t}{p^C_t C_t + p^V_t V_t} \right) \tag{14}
$$

$$
\begin{aligned}
V_t: \quad & \zeta_t z^V_t \varphi_V \left(\frac{V_t}{V_{t-1}} \right) + \beta E_t \left\{ \zeta_{t+1} z^V_{t+1} \tau'_V \left(\frac{V_{t+1}}{V_t} \right) \left(\frac{V_{t+1}}{V_t} \right)^2 \right\} \\
& = \lambda_t \varphi_M \left(\frac{M^{0,H}_t / P_t}{p^C_t C_t + p^V_t V_t} \right)
\end{aligned}
\tag{15}
$$

$$
K_{t+1}: \quad \zeta_t = \beta E_t \left\{ \zeta_{t+1} \left(1 - \delta^K \right) + \lambda_{t+1} \left[i^K_{t+1} u_{t+1} - \tau_u(u_{t+1}) \right] \right\} \tag{16}
$$

$$
u_t: \quad \lambda_t K_t \left[\tau'_u(u_t) - i^K_t \right] = 0 \tag{17}
$$

$$
D_t: \quad \lambda_t = \beta \left(1 + i^D_t \right) E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{18}
$$

$$
M^{0,H}_t: \quad \lambda_t \left[1 + \tau'_M \left(\frac{M^{0,H}_t / P_t}{p^C_t C_t + p^V_t V_t} \right) \right] = \beta E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \tag{19}
$$

$$W_t: \qquad 0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_W)^j \lambda_{t+j} h_{t+j} \frac{W_{t+j}}{P_{t+j}} \left(\pi^w_{t+j}\right)^{\psi} \tag{20}$$

$$\left\{ \left(\frac{\widetilde{W}_t}{W_t} \frac{\pi^w_t}{\pi^w_{t+j}} \right) - \frac{\psi}{\psi - 1} \frac{\eta_H z^H_{t+j} h^{\chi}_{t+j}}{\lambda_{t+j} W_{t+j}/P_{t+j}} \left(\frac{\widetilde{W}_t}{W_t} \frac{\pi^w_t}{\pi^w_{t+j}} \right)^{-\psi\chi} \right\}.$$

$$\lim_{t \to \infty} \beta^t D_t = 0. \tag{21}$$

Several comments are in order on these first order conditions.

First, we have used some auxiliary functions to alleviate notation. In (14) and (15) we have defined the function φ_M that gives the total effect on expenditure (i.e., including transaction cost related expenditures) of a marginal increase in absorption:[5]

$$\varphi_M(\varpi_t) \equiv 1 + \tau_M(\varpi_t) - \varpi_t \tau'_M(\varpi_t), \tag{22}$$
$$\varphi'_M(\varpi_t) = -\varpi_t \tau''_M(\varpi_t) < 0.$$

Notice that φ_M is decreasing in the money to absorption ratio ϖ_t and that the effect on expenditure generated by a marginal increase in ϖ_t is given by the increase in expenditure with the initial money/absorption ratio, $1 + \tau_M$, plus the increase due to the reduction in the money/absorption ratio, $\varpi_t(-\tau'_M(\varpi_t))$.

In analogous fashion, in (15) we have used the function φ_V defined as:

$$\varphi_V\left(\mu^V_t\right) \equiv 1 - \tau_V\left(\mu^V_t\right) - \mu^V_t \tau'_V\left(\mu^V_t\right),$$

(where $\mu^V_t \equiv V_t/V_{t-1}$ is the gross growth rate of V_t) which gives the increase in gross investment net of adjustment costs (but not of capital stock depreciation) resulting from a marginal increase in the rate of gross investment growth.[6]

(14) shows that in equilibrium the utility gain from a marginal increase in consumption, corrected for the habit related reduction in utility it is expected to generate next period (left side of the equality), equals the foregone marginal utility of real income it generates, including that which is related to transaction costs (given by $\varphi_M(.)$).

(15) shows that the loss in utility from marginally increasing gross investment (measured through the undiscounted shadow price of installed physical capital ζ_t and including investment adjustment costs) minus the discounted increase in utility it is expected to generate next period, equals the foregone marginal utility of real income it generates (including that which is related to transaction costs).

(16) states that the utility value of a marginal addition to installed capital equals the discounted expected utility value next period (corrected for capital depreciation) plus the discounted utility value of the net addition to rental income it is expected to generate.

(17) states that whenever the marginal utility of real income and the stock of physical capital are different from zero (which we assume is the case for all t), the equilibrium rate of utilization of physical capital is such that the marginal abnormal utilization cost equals the rental rate. Hence, this condition directly

[5] $\varphi_M(m/a)$ is the partial derivative of $[1 + \tau_M(m/a)]\, a$ with respect to a.
[6] $\varphi_V(V/V_{-1})$ is the partial derivative of $[1 - \tau_V(V/V_{-1})]\, V$ with respect to V.

determines the optimal intensity of utilization of physical capital as a function of the rental rate:

$$u_t = (\tau_u')^{-1}\left(i_t^K\right). \tag{23}$$

Inserting this expression in (4) gives the following auxiliary function for the net return from renting one unit of capital after taking abnormal utilization costs into account:

$$\Gamma^K\left(i_t^K\right) \equiv i_t^K (\tau_u')^{-1}\left(i_t^K\right) - \tau_u\left((\tau_u')^{-1}\left(i_t^K\right)\right). \tag{24}$$

(18) states that the loss in utility from marginally increasing the holding of deposits equals the discounted expected utility of the addition to real interest income it generates next period. And (19) states that the net loss of utility from marginally increasing the holding of currency after taking into account the reduction in transaction costs it generates, is equal to the discounted expected marginal utility of having it available tomorrow with its purchasing power corrected for inflation. Combining (18) and (19) yields:

$$-\tau_M'\left(\frac{M_t^{0,H}/P_t}{p_t^C C_t + p_t^V V_t}\right) = 1 - \frac{1}{1+i_t^D}, \tag{25}$$

which shows that the optimum stock of currency as a fraction of expenditure in consumption and investment is such that the reduction in transaction costs generated by a marginal increase in this ratio equals the opportunity cost of holding cash. Inverting $-\tau_M'$ gives the following demand function for cash as a vehicle for transactions (sometimes called "liquidity preference" function):

$$\frac{M_t^{0,H}}{P_t} = \mathcal{L}\left(1+i_t^D\right)\left[p_t^C C_t + p_t^V V_t\right], \tag{26}$$

where:

$$\mathcal{L}\left(1+i_t^D\right) \equiv (-\tau_M')^{-1}\left(1 - \frac{1}{1+i_t^D}\right)$$

$$\mathcal{L}'\left(1+i_t^D\right) = \left[-\tau_M''(.)\left(1+i_t^D\right)^2\right]^{-1} < 0.$$

From here on we replace the first order condition (19) by (26) and also use (26) to eliminate the household currency to absorption ratio wherever it appears through the use of the following auxiliary functions:

$$\widetilde{\varphi}_M(.) \equiv \varphi_M(\mathcal{L}(.)), \qquad \widetilde{\tau}_M(.) \equiv \tau_M(\mathcal{L}(.)). \tag{27}$$

Notice in (20) that since all households that can set their optimal wage in t make the same decision we have denoted the optimum wage rate $\widetilde{W}_t$. Hence, (8) and (5) imply the following law of motion for the aggregate wage rate (after assuming that the average wage rate of non-optimizers is the average overall wage level in $t-1$ indexed by wage inflation no matter when they optimized for the last time):

$$W_t^{1-\theta} = \alpha_W\left(W_{t-1}\pi_{t-1}^w\right)^{1-\theta} + (1-\alpha_W)\widetilde{W}_t^{1-\theta}. \tag{28}$$

Defining the real wage in terms of domestic goods and the relative wage between the optimizers and the general level:

$$w_t = \frac{W_t}{P_t}, \qquad \widetilde{w}_t = \frac{\widetilde{W}_t}{W_t},$$

the first order condition for W_t becomes:

$$\begin{aligned} 0 \;=\; & E_t \sum_{j=0}^{\infty} (\beta\alpha_W)^j \lambda_{t+j} h_{t+j} w_{t+j} \left(\pi_{t+j}^w\right)^{\psi} \\ & \left\{ \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w}\right) - \frac{\psi}{\psi-1} \frac{\eta_H z_{t+j}^H h_{t+j}^{\chi}}{\lambda_{t+j} w_{t+j}} \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w}\right)^{-\psi\chi} \right\}. \end{aligned} \tag{29}$$

And dividing through (28) by $W_{t-1}^{1-\theta}$ we get:

$$\left(\pi_t^w\right)^{1-\theta} = \alpha_W \left(\pi_{t-1}^w\right)^{1-\theta} + (1-\alpha_W)\left(\widetilde{w}_t \pi_t^w\right)^{1-\theta}, \tag{30}$$

which can be used to eliminate $\widetilde{w}_t$ from (29), leaving a dynamic equation in π_t^w. We refrain from doing so in the non-linear model, maintaining two dynamic equations for each inflation rate (wage and domestic, imported and exported goods) for the sake of clarity in the analysis of the steady state, but we eliminate this relative wage (and the corresponding relative prices for different types of goods) when we log-linearize the model.[7]

2.6. Domestic and imported consumption and investment goods

So far we have ignored the open economy attributes of consumption and investment, as well as the product differentiation within these classes. We now consider the household allocation of consumption and investment expenditures across these product classes and varieties. First we distinguish between domestic and imported consumption and investment goods. The consumption index we used in the household optimization problem is actually a constant elasticity of substitution (CES) aggregate consumption index of domestic and imported consumption goods:

$$C_t = \left(a_D^{\frac{1}{\theta_C}} \left(C_t^D\right)^{\frac{\theta_C - 1}{\theta_C}} + a_N^{\frac{1}{\theta_C}} \left(C_t^N\right)^{\frac{\theta_C - 1}{\theta_C}} \right)^{\frac{\theta_C}{\theta_C - 1}}, \qquad a_D + a_N = 1. \tag{31}$$

θ_C is the elasticity of substitution between domestic and imported consumption goods. And C_t^D and C_t^N are themselves CES aggregates of the domestic and imported (respectively) varieties of goods available:

$$C_t^D = \left(\int_0^1 C_t^D(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{32}$$

$$C_t^N = \left(\int_0^1 C_t^N(i)^{\frac{\theta_N-1}{\theta_N}} di \right)^{\frac{\theta_N}{\theta_N-1}}, \qquad \theta_N > 1. \tag{33}$$

[7] The detailed log-linearization of (29) and (30) is in Appendix 1.

θ and θ_N are the elasticities of substitution between varieties of domestic and imported goods in household expenditure, respectively. We assume that these elasticities hold for household expenditures in these goods whether they are for consumption or investment purposes. Total consumption expenditure is:

$$P_t^C C_t = P_t C_t^D + P_t^N C_t^N. \tag{34}$$

Then minimization of (34) subject to (31) for a given C_t, yields the following relations:

$$P_t = a_D^{\frac{1}{\theta_C}} P_t^C \left(\frac{C_t^D}{C_t}\right)^{-\frac{1}{\theta_C}} \tag{35}$$

$$P_t^N = a_N^{\frac{1}{\theta_C}} P_t^C \left(\frac{C_t^N}{C_t}\right)^{-\frac{1}{\theta_C}}. \tag{36}$$

Introducing these in (31) yields the consumption price index:

$$P_t^C = \left(a_D \left(P_t\right)^{1-\theta_C} + a_N \left(P_t^N\right)^{1-\theta_C}\right)^{\frac{1}{1-\theta_C}}. \tag{37}$$

Furthermore, it is readily seen that a_D and a_N in (31) are the shares of domestic and imported consumption in total consumption expenditures:

$$a_D = \frac{P_t C_t^D}{P_t^C C_t}, \qquad a_N = \frac{P_t^N C_t^N}{P_t^C C_t}. \tag{38}$$

With investment demand we proceed in exactly the same way. V_t is a CES aggregate investment index of domestic and imported investment goods:

$$V_t = \left(b_D^{\frac{1}{\theta_V}} \left(V_t^D\right)^{\frac{\theta_V-1}{\theta_V}} + b_N^{\frac{1}{\theta_V}} \left(V_t^N\right)^{\frac{\theta_V-1}{\theta_V}}\right)^{\frac{\theta_V}{\theta_V-1}}, \qquad b_D + b_N = 1, \tag{39}$$

where θ_V is the elasticity of substitution between domestic and imported investment goods, and V_t^D and V_t^N are CES aggregates of domestic and imported goods:

$$V_t^D = \left(\int_0^1 V_t^D(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \tag{40}$$

$$V_t^N = \left(V_t^N(i)^{\frac{\theta_N-1}{\theta_N}} di\right)^{\frac{\theta_N}{\theta_N-1}}, \qquad \theta_N > 1. \tag{41}$$

Then it follows that the investment price index is:

$$P_t^V = \left(b_D \left(P_t\right)^{1-\theta_V} + b_N \left(P_t^N\right)^{1-\theta_V}\right)^{\frac{1}{1-\theta_V}}. \tag{42}$$

and that the following relations hold:

$$P_t^V V_t = P_t V_t^D + P_t^N V_t^N.$$

$$P_t = b_D^{\frac{1}{\theta_V}} P_t^V \left(\frac{V_t^D}{V_t}\right)^{-\frac{1}{\theta_V}} \quad (43)$$

$$P_t^N = b_N^{\frac{1}{\theta_V}} P_t^V \left(\frac{V_t^N}{V_t}\right)^{-\frac{1}{\theta_V}}. \quad (44)$$

$$b_D = \frac{P_t V_t^D}{P_t^V V_t}, \qquad b_N = \frac{P_t^N V_t^N}{P_t^V V_t}. \quad (45)$$

Conditions (35), (36), (43), and (44) are necessary for the optimal allocation of household expenditures across domestic and imported goods in consumption and investment, respectively. Similarly, for the optimal allocation across varieties of domestic and imported goods within these classes, and using (32), (33), (40), and (41), the following conditions hold:

$$P_t(i) = P_t \left(\frac{C_t^D(i)}{C_t^D}\right)^{-\frac{1}{\theta_C}}$$

$$P_t^N(i) = P_t^N \left(\frac{C_t^N(i)}{C_t^N}\right)^{-\frac{1}{\theta_C}}.$$

$$P_t(i) = P_t \left(\frac{V_t^D(i)}{V_t}\right)^{-\frac{1}{\theta_V}}$$

$$P_t^N(i) = P_t^N \left(\frac{V_t^N(i)}{V_t^N}\right)^{-\frac{1}{\theta_V}}.$$

Finally, notice that (31) and (39) imply the following consumption and investment inflation rates:

$$\pi_t^C = \left[\frac{a_D}{a_D + a_N \left(p_{t-1}^N\right)^{1-\theta_C}} (\pi_t)^{1-\theta_C} + \left(1 - \frac{a_D}{a_D + a_N \left(p_{t-1}^N\right)^{1-\theta_C}}\right) \left(\pi_t^N\right)^{1-\theta_C}\right]^{\frac{1}{1-\theta_C}}$$

$$\pi_t^V = \left[\frac{b_D}{b_D + b_N \left(p_{t-1}^N\right)^{1-\theta_V}} (\pi_t)^{1-\theta_V} + \left(1 - \frac{b_D}{b_D + b_N \left(p_{t-1}^N\right)^{1-\theta_V}}\right) \left(\pi_t^N\right)^{1-\theta_V}\right]^{\frac{1}{1-\theta_V}},$$

the log-linear versions of which are:[8]

$$\widehat{\pi}_t^C = a_{PC} \widehat{\pi}_t^N + (1 - a_{PC}) \widehat{\pi}_t, \quad (46)$$

$$\widehat{\pi}_t^V = a_{PV} \widehat{\pi}_t^N + (1 - a_{PV}) \widehat{\pi}_t.$$

3. Domestic goods firms

3.1. Final domestic goods

There is perfect competition in the production (or bundling) of final domestic output Q_t, with the output of intermediate firms as inputs. A representative final domestic output firm uses the following CES technology:

$$Q_t = \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1 \quad (47)$$

[8] The coefficients a_{PC} and a_{PV} are defined in Appendix 2.

where θ is the elasticity of substitution between any two varieties of domestic goods and $Q_t(i)$ is the output of the intermediate domestic good i. Then the final domestic output representative firm solves the following problem each period:

$$\max_{Q_t(i)} P_t \left(\int_0^1 Q_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} - \int_0^1 P_t(i) Q_t(i) di, \tag{48}$$

the solution of which is:

$$Q_t(i) = Q_t \left(\frac{P_t(i)}{P_t} \right)^{-\theta}. \tag{49}$$

Introducing (49) in (47) and simplifying, it is readily seen that the domestic goods price index is:

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{50}$$

Also, introducing (49) into the cost part of (48) yields:

$$\int_0^1 P_t(i) Q_t(i) di = P_t Q_t.$$

3.2. Intermediate domestic goods

A continuum of monopolistically competitive firms produce intermediate domestic goods using labor, capital, and primary and imported inputs, with no entry or exit. They face perfectly competitive physical capital rental and primary commodities markets and perfectly competitive bundlers of import goods and labor types. The production function of firm i is:

$$Q_t(i) = \begin{cases} \epsilon_t K_t^{uD}(i)^{a^q} \left(z_t h_t(i) \right)^{b^q} \left(Q_t^{AD}(i) \right)^{c^q} N_t^D(i)^{1-a^q-b^q-c^q} - z_t F^D & \text{if positive} \\ 0 & \text{otherwise.} \end{cases} \tag{51}$$

ϵ_t and z_t are industry-wide productivity shocks. $K_t^{uD} \equiv u_t K_t^D$ is the flow of services rendered by the (hired) stock of capital K_t^D to domestic sector firms when used at the intensity u_t determined by the households that own them, Q_t^{AD} and N_t^D are the consumption of intermediate primary ("Agricultural") and imported inputs. $z_t F^D$ is a fixed cost that grows along with the economy and can be used to calibrate profits in the steady state[9]. $h_t(i)$ is a CES index of all the labor types:

$$h_t(i) = \left(\int_0^1 h_t(h,i)^{\frac{\psi-1}{\psi}} dh \right)^{\frac{\psi}{\psi-1}}, \tag{52}$$

where $h_t(h,i)$ is the amount of labor type h used by the domestic firm i. The production decision of i is subject to the demand function of final goods producers (49) and the price survival constraint, whereby the price it sets at t, $P_t(i)$ has a probability α of surviving (fully indexed) until the next period.

[9] Christiano, Eichenbaum and Evans (2001), for example, calibrate profits to zero.

3.3. Marginal cost and input demands

Extending the assumptions in CEE (2001) and in ALLV (2005) to the use of physical capital and primary and imported intermediate inputs, and allowing for randomness in the fractions of the different input costs that are bank financed, we assume that stochastic fractions ς_t^W, ς_t^K, ς_t^A, ς_t^N of the labor, capital rental, primary inputs and imported inputs bills, respectively, are financed by the domestic banking system. Let i_t^L be the bank nominal loan rate. At the end of period t-1 the firm formulates its demand for bank loans taking into account its expected financing needs in period t. Then we may write total variable cost as:

$$\Omega_t^K P_t i_t^K K_t^{uD}(i) + \Omega_t^W W_t h_t(i) + \Omega_t^A P_t^A Q_t^{AD}(i) + \Omega_t^N P_t^N N_t^D(i)$$

where P_t^A is the domestic currency price of primary goods, and[10]

$$\Omega_t^q = 1 + \varsigma_t^q i_{t-1}^L = \left[1 - \varsigma_t^q + \varsigma_t^q \left(1 + i_{t-1}^L\right)\right], \qquad q = K, W, A, N. \tag{53}$$

To maximize profits, the firm must minimize costs. It takes as given the wages $W_t(h)$ set by the different households. Consider first the minimization of total labor cost:

$$\int_0^1 W_t(h) h_t(h,i) dh \tag{54}$$

subject to a constant aggregate index or labor types (52). We call the Lagrange multiplier W_t. It does not depend on i since the problem is the same for all firms. Then the minimization results in i's inverse demand function for labor type h:

$$W_t(h) = W_t \left(\frac{h_t(h,i)}{h_t(i)}\right)^{-\frac{1}{\psi}}. \tag{55}$$

Defining the aggregate demand (over all firms) for labor of type h:

$$h_t(h) = \int_0^1 h_t(h,i) di,$$

and the aggregate demand (over all firms) for the labor bundle (over all households):

$$h_t = \int_0^1 h_t(i) di,$$

(55) implies the household labor demand (7) we used for the household problem. Furthermore, introducing (55) in (52) yields:

$$W_t = \left(\int_0^1 W_t(h)^{1-\psi} di\right)^{\frac{1}{1-\psi}},$$

confirming that the Lagrange multiplier is indeed the aggregate wage index as the notation implied. And introducing (55) in (54) yields a more convenient expression for the wage bill of firm i:

$$\int_0^1 W_t(h) h_t(h,i) dh = W_t h_t(i).$$

[10] The last expression in this equation is convenient for log-linearizing.

We now obtain factor and bank loan demands by solving the following cost minimization problem:

$$\min_{K_t^D(i),h_t(i),Q_t^{AD}(i),N_t^D(i)} \left\{\Omega_t^K P_t i_t^K K_t^{uD}(i) + \Omega_t^W W_t h_t(i) + \Omega_t^A P_t^A Q_t^{AD}(i) + \Omega_t^N P_t^N N_t^D(i)\right\}$$

subject to (51), where $Q_t(i)$ is given. The problem is the same for all firms, so we eliminate the firm index. The first order conditions are:

$$\Omega_t^K P_t i_t^K K_t^{uD} = a^q MC_t\left[Q_t + z_t F^D\right] \tag{56}$$

$$\Omega_t^W W_t h_t = b^q MC_t\left[Q_t + z_t F^D\right] \tag{57}$$

$$\Omega_t^A P_t^A Q_t^{AD} = c^q MC_t\left[Q_t + z_t F^D\right] \tag{58}$$

$$\Omega_t^N P_t^N N_t^D = (1 - a^q - b^q - c^q) MC_t\left[Q_t + z_t F^D\right], \tag{59}$$

where MC_t is the Lagrange multiplier. Adding these equations term by term shows that total variable cost is:

$$\Omega_t^K P_t i_t^K K_t^{uD} + \Omega_t^W W_t h_t + \Omega_t^A P_t^A Q_t^{AD} + \Omega_t^N P_t^N N_t^D = MC_t\left[Q_t + z_t F^D\right],$$

and that MC_t is indeed the nominal marginal cost. Furthermore, introducing the first order conditions and (53) in the production function (51) yields the following expressions for the nominal marginal cost:

$$\begin{aligned} MC_t &= \frac{1}{\kappa\epsilon_t\left(z_t\right)^{b^q}}\left(\Omega_t^K P_t i_t^K\right)^{a^q}\left(\Omega_t^W W_t\right)^{b^q}\left(\Omega_t^A P_t^A\right)^{c^q}\left(\Omega_t^N P_t^N\right)^{1-a^q-b^q-c^q}, \quad (60)\\ &= \frac{1}{\kappa\epsilon_t\left(z_t\right)^{b^q}} f_{MC}\left(1+i_{t-1}^L\right)\left(P_t i_t^K\right)^{a^q} W_t^{b^q}\left(P_t^A\right)^{c^q}\left(P_t^N\right)^{1-a^q-b^q-c^q} \end{aligned}$$

where we defined:

$$\kappa \equiv \left(a^q\right)^{a^q}\left(b^q\right)^{b^q}\left(c^q\right)^{c^q}\left(1-a^q-b^q-c^q\right)^{1-a^q-b^q-c^q},$$

and the auxiliary function:

$$\begin{aligned} f_{MC}\left(1+i_{t-1}^L\right) &\equiv \left[1-\varsigma_t^K+\varsigma_t^K\left(1+i_{t-1}^L\right)\right]^{a^q}\left[1-\varsigma_t^W+\varsigma_t^W\left(1+i_{t-1}^L\right)\right]^{b^q}\\ &\quad \left[1-\varsigma_t^A+\varsigma_t^A\left(1+i_{t-1}^L\right)\right]^{c^q}\left[1-\varsigma_t^N+\varsigma_t^N\left(1+i_{t-1}^L\right)\right]^{1-a^q-b^q-c^q},\\ f_{MC}'\left(1+i_{t-1}^L\right) &> 0. \end{aligned}$$

Hence, the (own) real marginal cost in the domestic sector is

$$mc_t \equiv \frac{MC_t}{P_t} = \frac{1}{\kappa\epsilon_t} f_{MC}\left(1+i_{t-1}^L\right)\left(i_t^K\right)^{a^q}\left(\frac{w_t}{z_t}\right)^{b^q}\left(p_t^A\right)^{c^q}\left(p_t^N\right)^{1-a^q-b^q-c^q}, \tag{61}$$

where

$$p_t^A \equiv \frac{P_t^A}{P_t}, \qquad p_t^N \equiv \frac{P_t^N}{P_t}$$

are the relative (domestic currency) prices of primary and imported goods, respectively, in terms of domestic goods. We refer to these relative prices as the SOE's primary and manufactured internal terms of trade, respectively.

Aggregate demand functions for h_t, K_t^{uD}, Q_t^{AD}, and N_t^D are obtained directly from (56)-(59) and (60). Notice that they all depend on the loan rate i_{t-1}^L, through the Ω_t^q $(q = W, K, A, N)$. Also, the resulting aggregate nominal demand for bank loans by firms in period t is:

$$L_t^F = E_t\{f_L\left(1+i_t^L\right) MC_{t+1}\left[Q_{t+1} + z_{t+1}F^D\right]\}, \tag{62}$$

where we defined the auxiliary function:

$$f_L\left(1+i_{t-1}^L\right) \equiv \frac{a^q \varsigma_t^K}{1+\varsigma_t^K i_{t-1}^L} + \frac{b^q \varsigma_t^W}{1+\varsigma_t^W i_{t-1}^L} + \frac{c^q \varsigma_t^A}{1+\varsigma_t^A i_{t-1}^L} + \frac{(1-a^q-b^q-c^q)\varsigma_t^N}{1+\varsigma_t^N i_{t-1}^L} \tag{63}$$
$$f_L'\left(1+i_{t-1}^L\right) < 0.$$

3.4. Sticky nominal price setting

As in the case of households, firms make pricing decisions taking the aggregate price and quantity indexes as parametric. Every period, each firm has a probability $1-\alpha$ of being able to set the optimum price for its specific type of good and whenever it can't optimize it adjusts its price by fully indexing to last period's overall rate of domestic inflation. Hence, when it can set its optimal price it must take into account that in any future period j there is a probability α^j that its price will be the one it sets today plus full indexation. Hence, the firm's price survival constraint states that the price it sets at t, $P_t(i)$ has a probability α^j of surviving (indexed) until period $t+j$:

$$P_{t+j}(i) = P_t(i)\pi_t\pi_{t+1}...\pi_{t+j-1} \equiv P_t(i)\Psi_{t,j}^p. \tag{64}$$

where $\Psi_{t,0}^p \equiv 1$. As in the case of wages (see (6)), we make use of the following identity:

$$\frac{P_t(i)}{P_{t+j}}\Psi_{t,j}^p = \frac{P_t(i)}{P_t}\frac{\pi_t}{\pi_{t+j}}. \tag{65}$$

Hence, we can express the firm's pricing problem as:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} \alpha^j \Lambda_{t,t+j}\left\{\frac{P_t(i)\Psi_{t,j}^p}{P_{t+j}} Q_{t+j}(i) - mc_{t+j}(i)\left[Q_{t+j}(i) + z_{t+j}F^D\right]\right\}$$

subject to

$$Q_{t+j}(i) = Q_{t+j}\left(\frac{P_t(i)\Psi_{t,j}^p}{P_{t+j}}\right)^{-\theta}.$$

$\Lambda_{t,t+j}$ is the pricing kernel used by firms for discounting, which is equal to households' intertemporal marginal rate of substitution in consumption between periods $t+j$ and t:

$$\Lambda_{t,t+j} = \beta^j \frac{U_{C,t+j}}{U_{C,t}} = \beta^j \frac{\lambda_{t+j}\widetilde{\varphi}_M\left(1+i_{t+j}^D\right)}{\lambda_t\widetilde{\varphi}_M\left(1+i_t^D\right)} \equiv \beta^j \frac{\overline{\Lambda}_{t+j}}{\overline{\Lambda}_t},$$

where $U_{C,t}$ is the household's marginal utility of consumption in t corrected for habit, and the second equality derives from (14) and (27).

The first order condition is the following (after dropping the firm index):

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \, \overline{\Lambda}_{t+j} Q_{t+j} \pi_{t+j}^{\theta} \left\{ \frac{\widetilde{P}_t}{P_t} \frac{\pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\}. \tag{66}$$

Since all optimizing firms make the same decision we call the optimum price $\widetilde{P}_t$. Hence, (50) and (64) imply the following law of motion for the aggregate domestic goods price index:

$$P_t^{1-\theta} = \alpha \left(P_{t-1}\pi_{t-1}\right)^{1-\theta} + (1-\alpha)\widetilde{P}_t^{1-\theta}. \tag{67}$$

Proceeding as we did with the wage inflation Phillips equation, we define the relative optimal to average domestic price:

$$\widetilde{p}_t = \frac{\widetilde{P}_t}{P_t},$$

and express the preceding equations as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \, \overline{\Lambda}_{t+j} Q_{t+j} \left(\pi_{t+j}\right)^{\theta} \left\{ \frac{\widetilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\},$$

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1-\alpha) \left(\widetilde{p}_t \pi_t\right)^{1-\theta}.$$

4. Primary goods producing firms

Firms in the primary sector use domestic goods, capital services and "land" (representing natural resources) to produce commodities. Land is assumed to be fixed in quantity, hence generating diminishing returns. We assume that there is a single homogenous primary good. Firms in this sector sell their output in the international market and also to domestic firms that use it as an input. They are price takers in factor and product markets.

Let the production function employed by firms in the primary sector be the following:

$$A_t = \left(Q_t^{DA}\right)^{\alpha_A} \left(u_t K_t^A\right)^{\beta_A}, \qquad \alpha_A + \beta_A < 1, \tag{68}$$

where A_t (for "Agriculture") is the amount produced, and Q_t^{DA} (not to be confused with Q_t^{AD}) and $u_t K_t^A$ are the amounts of domestic goods and physical capital services used as inputs in agriculture. These firms maximize profit

$$\Pi_t^A = P_t^A A_t - P_t Q_t^{DA} - P_t i_t^K u_t K_t^A$$

subject to (68). The first order conditions yield the factor demands:

$$Q_t^{DA} = \alpha_A p_t^A A_t \tag{69}$$

$$u_t K_t^A = \beta_A \frac{p_t^A}{i_t^K} A_t. \tag{70}$$

The domestic price of primary goods is merely the exogenous international price P_t^{**A} multiplied by the nominal exchange rate: $P_t^A = S_t P_t^{**A}$. Hence, the primary internal terms of trade (AITT) we used in (69) and (70) is

$$p_t^A \equiv \frac{P_t^A}{P_t} = \frac{S_t P_t^{**A}}{P_t} = e_t p_t^{**A}, \tag{71}$$

where we defined the SOE's real exchange rate (RER) and primary external terms of trade (AXTT):

$$e_t \equiv \frac{S_t P_t^{**N}}{P_t}, \qquad p_t^{**A} \equiv \frac{P_t^{**A}}{P_t^{**N}}.$$

The AXTT is exogenous in our model, as it is completely determined in the LRW. Using (23) in (70) shows that factor proportions in agriculture are determined by the physical capital rental rate:

$$\frac{Q_t^{DA}}{K_t^A} = \frac{\alpha_A}{\beta_A} i_t^K \left(\tau_u'\right)^{-1} \left(i_t^K\right).$$

Also, inserting the factor demand functions in the production function we see that optimal output varies directly with the RER and the AXTT, and inversely with the rental rate:

$$A_t = \left(\kappa_A \frac{\left(e_t p_t^{**A}\right)^{\alpha_A + \beta_A}}{\left(i_t^K\right)^{\beta_A}}\right)^{\frac{1}{1-\alpha_A-\beta_A}}, \qquad \kappa_A \equiv \left(\alpha_A\right)^{\alpha_A} \left(\beta_A\right)^{\beta_A}.$$

The exports of primary goods is whatever production is left over after satisfying the domestic sector's input demand given by (58):

$$X_t^A = A_t - c^q \frac{MC_t \left[Q_t + z_t F^D\right]}{\left(1 + \varsigma_t^A i_{t-1}^L\right) S_t P_t^{**A}} = A_t - c^q \frac{mc_t \left[Q_t + z_t F^D\right]}{\left(1 + \varsigma_t^A i_{t-1}^L\right) e_t p_t^{**A}}.$$

5. Foreign trade firms

We follow ALLV (2005) in allowing for an imperfect pass-through of exchange rate fluctuations by recurring to monopolistically competitive import and export firms that set prices with stickiness and local currency pricing. Because the "small open economy" concept is not always used with the same meaning, we explain what we mean by this below. We adopt the following notational conventions: 1) As in the case of the price of primary goods (P^{**A}), a double star ** as or within a superscript means that its value is determined in the "large rest of the world" (LRW) and hence the variable is exogenous in the model. 2) A single asterisk within a superscript means that it refers to prices in foreign currency (i.e., the average currency in the LRW). For example, the SOE's export firms set export prices P_t^{*MX} in the foreign currency (local currency pricing), and this is an endogenous variable.

5.1. Imported goods firms

Final imported goods

Perfectly competitive (trade) firms produce (or bundle) final imported goods using the output of monopolistically competitive intermediate imported goods producers. The representative firm in this sector uses the following CES technology:

$$N_t = \left(\int_0^1 N_t(i)^{\frac{\theta_N - 1}{\theta_N}} di\right)^{\frac{\theta_N}{\theta_N - 1}}, \qquad \theta_N > 1,$$

where θ_N is the elasticity of substitution between varieties of imported goods in consumption and investment as well as in their use as inputs for domestic goods firms. Maximizing profits (as in (48) for final domestic output firms) gives the demand function that the intermediate importer of good i faces:

$$N_t(i) = N_t \left(\frac{P_t^N(i)}{P_t^N} \right)^{-\theta_N},$$

where both price indexes are in the domestic currency. The resulting (domestic currency) price index for imported goods is:

$$P_t^N = \left(\int_0^1 P_t^N(i)^{1-\theta_N} di \right)^{\frac{1}{1-\theta_N}}, \tag{72}$$

and the import cost bill is:

$$\int_0^1 P_t^N(i) N_t(i) di = P_t^N N_t.$$

Intermediate imported goods

A continuum of monopolistically competitive firms generate intermediate imported goods. They buy a bundled final good abroad at the foreign price and turn it into differentiated goods to be sold in the domestic market in domestic currency (see ALLV (2005)). They purchase the bundled final good at the price $S_t P_t^{**N}$, where P_t^{**N} is the foreign currency price index of the imported bundle (which we assume differs from the LRW's "domestic" price index P_t^{**}) and S_t is the nominal exchange rate (pesos per unit of foreign currency). Notice that $S_t P_t^{**N}$ is thus the marginal cost for these firms. Their pricing (in the domestic currency) follows the same setup we used for firms producing domestic intermediate goods, with a probability $1 - \alpha_N$ of optimal price setting and full indexation when they can't optimize price. According to the price survival constraint, the price $P_t^N(i)$ the firm sets at t has a probability α_N^j of surviving (indexed) until $t + j$:

$$P_{t+j}^N(i) = P_t^N(i) \pi_t^N \pi_{t+1}^N ... \pi_{t+j-1}^N \equiv P_t^N(i) \Psi_{t,j}^N, \qquad \left(\Psi_{t,0}^N \equiv 1 \right). \tag{73}$$

When the firm optimizes it takes into account that there is a probability α_N^j that the demand for its good in $t + j$ will be:

$$N_{t+j}(i) = N_{t+j} \left(\frac{P_t^N(i) \Psi_{t,j}^N}{P_{t+j}^N} \right)^{-\theta_N}. \tag{74}$$

Hence, they solve:

$$\max_{P_t^N(i)} E_t \sum_{j=0}^{\infty} \alpha_N^j \Lambda_{t,t+j} N_{t+j}(i) \left\{ \frac{P_t^N(i) \Psi_{t,j}^N}{P_{t+j}^N} - \frac{S_{t+j} P_{t+j}^{**N}}{P_{t+j}^N} \right\}$$

subject to (74). After eliminating the firm index, the resulting first order condition is:

$$0 = E_t \sum_{j=0}^{\infty} (\beta \alpha_N)^j \, \overline{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\widetilde{P}_t^N}{P_t^N} \frac{\pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{S_{t+j} P_{t+j}^{**N}}{P_{t+j}^N} \right\}. \tag{75}$$

Since all optimizing firms make the same decision, we call the optimal import price $\widetilde{P}_t^N$. Hence (72) and (73) imply the following law of motion for the aggregate domestic currency import price index:

$$\left(P_t^N\right)^{1-\theta_N} = \alpha_N \left(P_{t-1}^N \pi_{t-1}^N\right)^{1-\theta_N} + (1-\alpha_N)\left(\widetilde{P}_t^N\right)^{1-\theta_N}. \tag{76}$$

Using our definitions of e_t and p_t^N, we can express the preceding equations as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_N)^j\, \overline{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\widetilde{p}_t^N \pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{e_{t+j}}{p_{t+j}^N} \right\}$$

$$\left(\pi_t^N\right)^{1-\theta_N} = \alpha_N \left(\pi_{t-1}^N\right)^{1-\theta_N} + (1-\alpha_N)\left(\widetilde{p}_t^N \pi_t^N\right)^{1-\theta_N},$$

where

$$\widetilde{p}_t^N \equiv \frac{\widetilde{P}_t^N}{P_t^N}$$

is the relative price between optimized and overall imported goods. Notice that

$$\frac{e_t}{p_t^N} = \frac{S_t P_t^{**N}}{P_t^N}$$

measures the deviation (whenever it differs from 1) from the Law of one Price for imported goods.

5.2. Manufactured exports firms

There are two types of exporting firms in the paper: those that export commodities (which we have already addressed), and those that export domestic (i.e. manufactured) goods. This section addresses the latter firms, which are sticky price setters that use local currency pricing.

Final manufactured exports Each of a continuum of intermediate exporting firms purchases the final domestic good at its price P_t (which is hence its marginal cost) and differentiates it to sell in different foreign markets with local currency pricing. The goods are purchased by a representative perfectly competitive final exporting firm that has a CES technology:

$$X_t^M = \left(\int_0^1 X_t^M(i)^{\frac{\theta^*-1}{\theta^*}} di \right)^{\frac{\theta^*}{\theta^*-1}}, \qquad \theta^* > 1,$$

where θ^* is the elasticity of substitution in the rest of the world for the imported goods that originate in the SOE. Maximizing profit, as in the previous cases, gives the demand function each intermediate exporting firm faces from the final exporters:

$$X_t^M(i) = X_t^M \left(\frac{P_t^{*MX}(i)}{P_t^{*MX}} \right)^{-\theta^*}. \tag{77}$$

Notice that the price $P_t^{*MX}(i)$ is in foreign currency. The resulting foreign currency price index for exported goods is:

$$P_t^{*MX} = \left(\int_0^1 P_t^{*MX}(i)^{1-\theta^*} di \right)^{\frac{1}{1-\theta^*}}, \tag{78}$$

and the foreign currency cost bill for the representative final exporting (bundling) firm is:

$$\int_0^1 P_t^{*MX}(i) X_t^M(i) di = P_t^{*MX} X_t^M.$$

Notice that in the demand function for exports (77), X_t^M is the rest of the world's imports of manufactured goods from the SOE (which we can alternatively write as N_t^{*M}) and P_t^{*MX} is the rest of the world's aggregate manufactured import price from the SOE. Hence, we can alternatively write (77) as:

$$X_t^M(i) = N_t^{*M} \left(\frac{P_t^{*MX}(i)}{P_t^{*MX}} \right)^{-\theta^*}.$$

We further assume that the rest of the world's aggregate manufactured imports from the SOE X_t^M is related to its output (Y_t^{**}) and its output price index (P_t^{**}) by:

$$N_t^{*M} \equiv X_t^M = Y_t^{**} \left(\frac{P_t^{*MX}}{P_t^{**}} \right)^{-\theta^{**}},$$

where θ^{**} is a parameter different from θ^*. Notice that the relative price in the last expression can be written as:

$$\frac{P_t^{*MX}}{P_t^{**}} = \frac{P_t^{*MX}}{P_t^{**N}} \frac{P_t^{**N}}{P_t^{**}} = p_t^{*MX} p_t^{**X}, \tag{79}$$

where we defined the SOE's manufactured external terms of trade (MXTT) and the LRW's (manufactured) export to "domestic" relative price:

$$p_t^{*MX} \equiv \frac{P_t^{*MX}}{P_t^{**N}}, \qquad p_t^{**X} \equiv \frac{P_t^{**N}}{P_t^{**}}.$$

The first of these relative prices is endogenous in our model due to exporters' price setting, as we further elaborate below, and the second is exogenous. Notice that we do not assume that the law of one price prevails in the long run (non-stochastic steady state). In the context of monopolistic competition, any good produced by a firm in the domestic economy is not produced by any other firm in the world. Hence, the law of one price only means that any domestic good i must be sold in the rest of the world at the same price it sells domestically after expressing it in foreign currency: $P_t^{*MX}(i) = P_t(i)/S_t$, and that any good i produced in the LRW must be sold domestically at the price $P_t^N(i) = S_t P_t^{**N}(i)$. We see no reason to assume such lack of market segmentation for manufactured goods, even in the model's long run (see Kollman (2001)).

Intermediate manufactured exports Intermediate manufactured export firms set prices in foreign currency taking the foreign price and quantity indexes P_t^{**}, Y_t^{**}, as parameters. The local (foreign) currency pricing of intermediate manufactured export goods firms follows the same setup we used previously, with a probability $1 - \alpha_X$ of optimal price setting and full indexation when they can't change price.

Hence, according to their price survival constraint they face a probability α_X^j of having the price they set at t survive (indexed) until $t+j$:

$$P_{t+j}^{*MX}(i) = P_t^{*MX}(i)\pi_t^{*MX}\pi_{t+1}^{*MX}...\pi_{t+j-1}^{*MX} \equiv P_t^{*MX}(i)\Psi_{t,j}^{*X}. \tag{80}$$

Hence, when taking (77) as a constraint, they must consider that there is a probability α_X^j that their demand in $t+j$ will be:

$$X_{t+j}^M(i) = X_{t+j}^M\left(\frac{P_t^{*MX}(i)\Psi_{t,j}^{*X}}{P_{t+j}^{*MX}}\right)^{-\theta^*}. \tag{81}$$

When they can set their optimal price they solve:

$$\max_{P_t^{*X}(i)} E_t \sum_{j=0}^{\infty} \alpha_X^j \Lambda_{t,t+j} X_{t+j}^M(i)\left\{\frac{P_t^{*MX}(i)\Psi_{t,j}^{*X}}{P_{t+j}^{*MX}} - \frac{P_{t+j}}{S_{t+j}P_{t+j}^{*MX}}\right\}$$

subject to (81). The first order condition is:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_X)^j\,\overline{\Lambda}(X_{t+j}^M \pi_{t+j}^{*X})^{\theta^*}\left\{\frac{\widetilde{P}_t^{*MX}}{P_t^{*MX}}\frac{\pi_t^{*MX}}{\pi_{t+j}^{*MX}} - \frac{\theta^*}{\theta^*-1}\frac{P_{t+j}}{S_{t+j}P_{t+j}^{*MX}}\right\}.$$

Since all optimizing firms make the same decision we call the optimal foreign currency manufactured export price $\widetilde{P}_t^{*MX}$, and (78) and (80) imply the following law of motion for the aggregate price level of exports:

$$\left(P_t^{*MX}\right)^{1-\theta^*} = \alpha_X\left(P_{t-1}^{*MX}\pi_{t-1}^{*MX}\right)^{1-\theta^*} + (1-\alpha_X)\left(\widetilde{P}_t^{*MX}\right)^{1-\theta^*}. \tag{82}$$

To simplify these expressions as we did previously, notice first that the own marginal cost of intermediate export firms is the inverse of the product of the SOE's RER and its external terms of trade:

$$\frac{P_t}{S_t P_t^{*MX}} = \frac{1}{e_t p_t^{*MX}}. \tag{83}$$

Next, we define the relative price between optimizing and overall export prices:

$$\widetilde{p}_t^{*MX} \equiv \frac{\widetilde{P}_t^{*MX}}{P_t^{*MX}},$$

and express the dynamic equations for export prices as:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_X)^j\,\overline{\Lambda}_{t+j} X_{t+j}^M (\pi_{t+j}^{*MX})^{\theta^*}\left\{\frac{\widetilde{p}_t^{*MX}\pi_t^{*MX}}{\pi_{t+j}^{*MX}} - \frac{\theta^*}{\theta^*-1}\frac{1}{e_{t+j}p_{t+j}^{*MX}}\right\}.$$

$$\left(\pi_t^{*MX}\right)^{1-\theta^*} = \alpha_X\left(\pi_{t-1}^{*MX}\right)^{1-\theta^*} + (1-\alpha_X)\left(\widetilde{p}_t^{*MX}\pi_t^{*MX}\right)^{1-\theta^*}.$$

Notice that (83) measures the deviation (whenever it differs from 1) from the Law of one Price for manufactured export goods (see Kollman (2001)). In general we have manufacturing exporters (both in the SOE and in the LRW) exerting their monopoly power even in the steady state. Hence, as we see further below, in the steady state we have both $e/p^N < 1$ and $ep^{*MX} > 1$.

SOE			LRW	
Domestic Currency			Foreign Currency	
	Relative Prices	Monetary Prices	Monetary Prices	Relative Prices
RER	$e=SP^{**N}/P$			$e^{*}= P/ SP^{**}= p^{**X} / e$
		P	P^{**}	
MITT	$p^{N}= P^{N}/P$			$p^{N*}= P^{*MX} / P^{**}= p^{*MX}p^{**X}$
		P^{N}	P^{*MX}	
SDE's MXTT				$p^{*MX}= P^{*MX} / P^{**N}$
		SP^{**N}	P^{**N}	
SDE's AXTT				$p^{**A}= P^{**A} / P^{**N}$
		$P^{A} =SP^{**A}$	P^{**A}	
AITT	$p^{A}= P^{A}/P$			$p^{A**}= P^{**A} / P^{**} = p^{**A}\ p^{**X}$

6. A review of some important relative prices

Figure 1 summarizes the international pricing of the model. The SOE's and the LRW's main monetary price indexes (P_t, P_t^N, and P_t^{**}, P_t^{*MX}, P_t^{**N}, P_t^{**A}, respectively) are shown in the two central columns. For each there is a domestic price index and an imported manufactured goods price index, each in terms of its own currency. The two outer columns show the main relative prices. In each economy, the relative price between (manufactured) imported and domestic price indexes defines its manufactured internal terms of trade (MITT): p_t^N and p_t^{N*}, respectively. In the LRW, however, we also distinguish a manufactured export price index P_t^{**N} and a primary goods price index P_t^{**A}, both of which differ from its "domestic" price index P_t^{**}. Hence, there are additional relative prices p_t^{*MX} and p_t^{**A} for its manufactured import and export goods in terms of "domestic" goods, all expressed in "domestic" currency (i.e. foreign currency). The first is the SOE's manufactured external terms of trade (MXTT), and the second is its primary external terms of trade (AXTT).

In each economy a certain price index is converted into the corresponding export price index through local currency pricing (i.e. pricing in the partner's currency) and this is the trade partner's manufactured import price index. The solid arrows indicate the local currency pricing of exporters. Also, in each economy the RER is defined as the relative price between the partner's manufactured export bundle, converted to the domestic (or "domestic") currency through the nominal exchange rate, and the domestic (or "domestic") bundle. Hence, the SOE's RER is the relative price between imported goods as they are purchased in the LRW by importers and domestic goods, both expressed in a common currency:

$$e_t \equiv \frac{S_t P_t^{**N}}{P_t}. \tag{84}$$

Here, P_t^{**N} is the rest of the world's export price index and, hence, is an exogenous variable in our model. With the same definition, the LRW's RER turns out to be its export to domestic relative price ($p_t^{**X} \equiv P_t^{**N}/P_t^{**}$) divided by the SOE's RER:

$$e_t^* \equiv \frac{P_t/S_t}{P_t^{**}} = \frac{P_t}{S_t P_t^{**N}} \frac{P_t^{**N}}{P_t^{**}} = \frac{p_t^{**X}}{e_t}.$$

Since the SOE is insignificant in size in relation to the LRW, its actions have no influence in the LRW's allocation of resources. We do not need e_t^* in the model.

The SOE's manufactured internal terms of trade (MITT) is the relative price between imported and domestic goods as faced by households and domestic firms and is an endogenous variable:

$$p_t^N \equiv \frac{P_t^N}{P_t}. \tag{85}$$

It is a ratio between two domestic currency prices. With a similar definition, the LRW's MITT is a ratio between the price indexes of its manufactured imported goods (i.e. the SOE's manufactured exports) and its "domestic" goods (both in foreign currency):

$$p_t^{N*} \equiv \frac{P_t^{*MX}}{P_t^{**}} = p_t^{*MX} p_t^{**X}, \tag{86}$$

and, as in (79), can be expressed as the product of the SOE's MXTT (which is an endogenous variable) and the LRW's export to domestic relative price index (which is exogenous). Again, we do not need p_t^{N*} in the model. In a similar fashion we have the primary internal terms of trade (AITT) for the SOE economy and the LRW, respectively:

$$p_t^A \equiv \frac{P_t^A}{P_t} = e_t p_t^{**A} \tag{87}$$

$$p_t^{**A} \equiv \frac{P_t^{**A}}{P_t^{**}} = p_t^{**A} p_t^{**X}. \tag{88}$$

Although we have chosen to call e_t the real exchange rate, an alternative way of defining our terms would be to call this variable the imports real exchange rate and

$$e_t p_t^{**X} = \frac{S_t P_t^{*MX}}{P_t} \quad \text{and} \quad e_t p_t^{**A} = \frac{S_t P_t^{**A}}{P_t}, \tag{89}$$

the manufactured exports and primary goods real exchange rates, respectively. We have preferred to have a single variable defined as real exchange rate.

7. Banks

We assume that there is a competitive banking industry, with no entry or exit. Banks are owned by households, and are price takers in financial markets. They obtain funds in the international market B_t^{*B} (a constant fraction γ^{FX} of which they hold as vault cash in foreign exchange), supply one period deposit facilities to households D_t, and use the proceeds to supply one period loans to firms and the government $\left(L_t = L_t^F + L_t^G\right)$, lend (or borrow) in the interbank market, purchase (or sell) Central Bank bonds B_t^{CB}, and hold peso vault cash $M_t^{0,B}$ as well as regulatory reserves R_t^B in the Central Bank. Any interbank loans cancel out and profits that arise from period t-1 operations are distributed to owners in period t, so the balance sheet constraint for the representative bank is:

$$L_t + B_t^{CB} + M_t^{0,B} + R_t^B = D_t + \left(1 - \gamma^{FX}\right) S_t B_t^{*B}. \tag{90}$$

We assume that vault cash is a (technical) fraction γ^B of deposits, and that interbank deposits are perfect substitutes for Central Bank bonds (so they earn the same interest rate i_t). Since we also assume that the Central Bank does not pay interest on regulatory reserves, banks keep these at the minimum, which is assumed to be a proportion γ_t^R of deposits. γ_t^B and γ_t^R are exogenous and may be stochastic processes. Hence, (90) is equivalent to:

$$L_t + B_t^{CB} = (1 - \gamma_t^B - \gamma_t^R) D_t + \left(1 - \gamma^{FX}\right) S_t B_t^{*B} \tag{91}$$

We assume that interest on banks' foreign debt is paid out in the following period, just before profits are distributed to owners. Since banks' business is (assumed to be) in domestic currency, they face exchange rate uncertainty. For every unit of foreign currency they repay they must expect to have pesos in the amount of

$$\delta_{t+1}^e (1 + i_t^B),$$

where

$$\delta_{t+1} = \frac{S_{t+1}}{S_t}$$

and δ_{t+1}^e, are the actual and expected rate of nominal peso depreciation, respectively. To add some additional inertia, we assume that a fraction β^B of banks has rational expectations and that the remaining fraction has simple static expectations by which

$$\delta_{t+1}^e = \delta_t.$$

Except for this heterogeneity in expectations, all banks are the same. Hence, on average the expected rate of nominal depreciation by banks is:

$$\delta_{t+1}^e = \beta^B E_t \delta_{t+1} + \left(1 - \beta^B\right) \delta_t.$$

We also assume that banks must pay a premium on the international riskless rate i_t^{**} for the funds they obtain abroad. Since we do not model the rest of the world, the risk premium (function) is exogenously given. It has an exogenous component ϕ_t^{**B} (a risk premium shock) as well as an endogenous component $p_B(.)$ that is an increasing convex function of the trend adjusted (individual) bank foreign debt (see Turnovsky (2000) and Schmitt-Grohé and Uribe (2003))[11]. Individual banks thus fully internalize the fact that their individual foreign debt decision determines the foreign currency interest rate they face, which is:

$$1 + i_t^B = (1 + i_t^{**}) \left[1 + \phi_t^{**B} + p_B\left(\frac{S_t B_t^{*B}}{P_t z_t}\right)\right], \tag{92}$$

where z_t is the permanent productivity shock in the domestic output sector that we use for detrending, and we assume $p_B' > 0$ and $p_B'' > 0$.

[11] We assume foreign creditors do not net out the dollar vault cash that banks hold when assessing risk.

Banks have a real cost function that depends on the real deposit and loan creating activities of the bank. We assume this cost function is quadratic and implies that there are economies of scope between lending and deposit taking activities (see Freixas and Rochet (1997), chapter 3). Specifically, we assume the following real cost function:

$$\begin{aligned} C^B_{t+1} &= C^B(L_t, D_t, z_t P_t) = \\ &= \frac{1}{2}\left[a^B_L\left(\frac{L_t}{z_t P_t}\right)^2 + a^B_D\left(\frac{D_t}{z_t P_t}\right)^2 - 2a^B_0\left(\frac{L_t}{z_t P_t}\right)\left(\frac{D_t}{z_t P_t}\right)\right] \\ &= \frac{1}{2}\left[\frac{a^B_L L_t{}^2 + a^B_D D_t{}^2 - 2a^B_0 L_t D_t}{(z_t P_t)^2}\right], \qquad \left(a^B_L > a^B_0 > 0, a^B_D > a^B_0, \right). \end{aligned} \tag{93}$$

The representative bank maximizes expected profit each period:

$$E_t \Pi^B_{t+1} = i^L_t L_t + i_t B^{CB}_t - i^D_t D_t - \delta^e_{t+1} i^B_t S_t B^{*B}_t \left(1 - \gamma^{FX}\right) - P_t z_t C^B_{t+1}$$

subject to its balance sheet constraint (91), its supply of foreign funds constraint (92), and its cost function (93). The solution to this program gives the supply of loans and deposits in terms of the loan margin $i^L_t - i_t$ and the deposit margin $(1 - \gamma^B_t - \gamma^R_t)i_t - i^D_t$, and the optimal amount of foreign funding in the form of a "risk-adjusted uncovered interest parity" relation:

$$L^S_t = \frac{z_t P_t}{a^B}\{a^B_D\left(i^L_t - i_t\right) + a^B_0\left[(1 - \gamma^B_t - \gamma^R_t)i_t - i^D_t\right]\} \tag{94}$$

$$D^S_t = \frac{z_t P_t}{a^B}\{a^B_L\left[(1 - \gamma^B_t - \gamma^R_t)i_t - i^D_t\right] + a^B_0\left(i^L_t - i_t\right)\} \tag{95}$$

$$i_t = \left[\beta^B E_t \delta_{t+1} + \left(1 - \beta^B\right)\delta_t\right]\left[\left(1 + i^{**}_t\right)\varphi_B\left(\frac{S_t B^{*B}_t}{P_t z_t}\right) - 1\right], \tag{96}$$

where we defined the following auxiliary function for the multiplicative gross risk adjustment to the uncovered interest parity:

$$\varphi_B\left(\frac{S_t B^{*B}_t}{P_t z_t}\right) \equiv 1 + \phi^{**B}_t + p_B\left(\frac{S_t B^{*B}_t}{P_t z_t}\right) + \left(\frac{S_t B^{*B}_t}{P_t z_t}\right) p'_B\left(\frac{S_t B^{*B}_t}{P_t z_t}\right). \tag{97}$$

Notice that the assumptions made on a^B_L, a^B_D, and a^B_0, imply:

$$a^B \equiv a^B_L a^B_D - \left(a^B_0\right)^2 > 0.$$

Given our assumptions on $p_B(.)$, the positiveness of a^B is necessary and sufficient to ensure that the first order conditions yield maximum profits. The resulting optimal bank cost and profit are:

$$\begin{aligned} C^B_{t+1} &= \frac{1}{2a^B}\{a^B_D\left[i^L_t - i_t\right]^2 + a^B_L\left[(1 - \gamma^B_t - \gamma^R_t)i_t - i^D_t\right]^2 \\ &\quad + 2a^B_0\left[i^L_t - i_t\right]\left[(1 - \gamma^B_t - \gamma^R_t)i_t - i^D_t\right]\}. \end{aligned}$$

$$\frac{\Pi^B_{t+1}}{z_t P_t} = C^B_{t+1} + \left(\frac{S_t B^{*B}_t}{P_t z_t}\right)^2 p'_B\left(\frac{S_t B^{*B}_t}{P_t z_t}\right).$$

Given L^S_t, D^S_t, and B^{*B}_t, the aggregate bank demand for Central Bank bonds is given by the aggregate bank balance sheet constraint:

$$B^{CB,D}_t = (1 - \gamma^B_t - \gamma^R_t)D^S_t + \left(1 - \gamma^{FX}\right) S_t B^{*B}_t - L^S_t. \tag{98}$$

8. The public sector

The public sector is made up of the Government and the Central Bank.

8.1. The Government

The Government issues foreign currency denominated bonds in the international markets, obtains loans from banks and pays interest on these bonds and loans, spends on goods, and collects taxes. We assume that fiscal policy consists of exogenous paths for nominal lump-sum tax collection (T_t), nominal bank loans (L_t^G), and real expenditures (G_t). The Government finances any resulting deficit by issuing foreign currency denominated bonds (B_t^{*G}). The exogenous paths are assumed to be compatible with a finite non-stochastic steady state for government debt. To hold foreign currency denominated government bonds, foreign investors charge a risk premium over the risk-free foreign interest rate (i_t^{**}). As in the case of banks, the risk premium (function) is exogenously given and is assumed to have an exogenous stochastic component (an external financing shock) and an endogenous component which is an increasing function of the trend adjusted public sector net foreign liability. Hence the gross interest rate on the government's foreign debt is:

$$1 + i_t^G = (1 + i_t^{**}) \left[1 + \phi_t^{**G} + p_G \left(\frac{S_t \left(B_t^{*G} - R_t^{*CB} \right)}{P_t z_t} \right) \right]. \tag{99}$$

where $p_G' > 0$, and R_t^{*CB} is the Central Bank's international reserves.

The Government flow budget constraint is:

$$S_t B_t^{*G} + L_t^G = P_t G_t - T_t + (1 + i_{t-1}^G) S_t B_{t-1}^{*G} + \left(1 + i_{t-1}^L\right) L_{t-1}^G. \tag{100}$$

8.2. The Central Bank

The Central Bank issues currency (M_t^0), domestic currency bonds $\left(B_t^{CB}\right)$, and debt certificates to banks for non-remunerated reserves $\left(R_t^B\right)$, and holds international reserves $\left(R_t^{*CB}\right)$ in the form of foreign currency denominated riskless bonds issued abroad. We assume that Central Bank bonds are only held by domestic banks. The (flow) budget constraint of the Central Bank is:

$$M_t^0 + R_t^B + B_t^{CB} - S_t R_t^{*CB} = M_{t-1}^0 + R_{t-1}^B + (1 + i_{t-1}) B_{t-1}^{CB} - (1 + i_{t-1}^*) S_t R_{t-1}^{*CB} \tag{101}$$

$$= \left[M_{t-1}^0 + R_{t-1}^B + B_{t-1}^{CB} - S_{t-1} R_{t-1}^{*CB} \right] - \left[i_{t-1}^* S_t R_{t-1}^{*CB} + (S_t - S_{t-1}) R_{t-1}^{*CB} \right] - i_{t-1} B_{t-1}^{CB}$$

The second term in square brackets after the last equality is the Central Bank's quasi-fiscal surplus. It includes interest earned and capital gains on international reserves minus the interest paid on its bonds. We assume that the Central Bank transfers its quasi-fiscal surplus (or deficit) to the Government every period. Hence, the Central Bank's balance sheet "constraint" is always preserved:

$$M_t^0 + R_t^B = S_t R_t^{*CB} - B_t^{CB}. \tag{102}$$

In our model, this equation implicitly defines the Central Bank's backing of its monetary base ($M_t^0 + R_t^B$) with its international reserves net of its bond liabilities. The Central Bank supplies whatever monetary base is demanded by households and banks, and can influence these supplies by changing R_t^{*CB} or B_t^{CB}, i.e. intervene in the foreign exchange market or in the interbank cum Central Bank bond market.

Adding (100) and (101) term by term gives the consolidated public sector budget constraint:

$$M_t^0 + R_t^B + B_t^{CB} + S_t\left(B_t^{*G} - R_t^{*CB}\right) + L_t^G = P_t G_t - T_t + \left(1 + i_{t-1}^L\right) L_{t-1}^G \quad (103)$$
$$+M_{t-1}^0 + R_{t-1}^B + (1 + i_{t-1})B_{t-1}^{CB} + (1 + i_{t-1}^G)S_t B_{t-1}^{*G} - (1 + i_{t-1}^{**})S_t R_{t-1}^{*CB}.$$

Using (102) and its equivalent for $t-1$, this expression reduces to:

$$S_t B_t^{*G} = (1 + i_{t-1}^G)S_t B_{t-1}^{*G} - QF_t - GD_t, \quad (104)$$

where QF_t is the Central Bank's quasi-surplus and GD_t is the Government's domestic surplus:

$$\begin{aligned} QF_t &\equiv \left[i_{t-1}^{**} + (1 - S_{t-1}/S_t)\right] S_t R_{t-1}^{*CB} - i_{t-1}B_{t-1}^{CB} \\ GD_t &\equiv T_t - P_t G_t + L_t^G - \left(1 + i_{t-1}^L\right) L_{t-1}^G \end{aligned}$$

The Government sells foreign currency bonds in international capital markets to the extent that the sum of its capital repayments and interest payments on these bonds exceeds the sum of the domestic currency value of the Central Bank's quasi-surplus and the Government's domestic surplus. The Central Bank's quasi-surplus is composed of interest earnings on international reserves, capital gains on international reserves due to changes in the exchange rate net of interest payments on its stock peso bonds.

9. Market clearing equations, GDP, and the balance payments

9.1. Market clearing

In the physical capital rental market, market clearing implies that the household supply of physical capital at the optimal intensity level equals domestic (56) and primary (70) sector demands:

$$(\tau_u')^{-1}\left(i_t^K\right) K_t = a^q \frac{mc_t}{\left(1 + \varsigma_t^W i_{t-1}^L\right) i_t^K}\left[Q_t + z_t F^D\right] + \beta_A \frac{e_t p^{**A}}{i_t^K} A_t. \quad (105)$$

In the labor market, the household supply of labor h_t equals domestic firms' demand (57):

$$h_t = b^q \frac{mc_t}{\left(1 + \varsigma_t^W i_{t-1}^L\right) w_t}\left[Q_t + z_t F^D\right]. \quad (106)$$

In the loan market, bank loan supply (given by (94), which we maintain as a separate equation) equals loan demand by firms (62) and the government:

$$\frac{L_t}{P_t} = E_t\left\{f_L\left(1 + i_t^L\right) mc_{t+1}\left[Q_{t+1} + z_{t+1}F^D\right]\right\} + \frac{L_t^G}{P_t}. \quad (107)$$

Notice that in the last three equations mc_t is given by (61).

In the deposit market, household deposit demand equals bank deposit supply (95):

$$\frac{D_t}{P_t} = \left(z_t/a^B\right)\left\{a_L^B\left[(1 - \gamma_t^B - \gamma_t^R)i_t - i_t^D\right] + a_0^B\left(i_t^L - i_t\right)\right\} \quad (108)$$

In the interbank cum Central Bank bond market, interbank loans cancel out and Central Bank supply B_t^{CB} equals aggregate bank demand (98):

$$B_t^{CB} = (1-\gamma_t^B-\gamma_t^R)D_t + \left(1-\gamma^{FX}\right) S_t B_t^{*B} - L_t, \tag{109}$$

where Central Bank supply is derived from its balance sheet constraint (102), which we maintain as a separate equation.

In the cash market, the supply of currency equals household (26) and bank demand:

$$M_t^0 = \mathcal{L}\left(1+i_t^D\right)\left[P_t^C C_t + P_t^V V_t\right] + \gamma_t^B D_t. \tag{110}$$

In the domestic goods market, the output of domestic firms Q_t must satisfy final demand from households and the Government, as well as intermediate demand from the manufactured exports, primary and banking sectors, and the goods used up pertaining to abnormal capital utilization costs and transaction costs:

$$\begin{aligned} Q_t &= a_D p_t^C C_t + b_D p_t^V V_t + G_t + X_t^M + Q_t^{DA} + z_t C_{t+1}^B \\ &+\tau_u((\tau_u')^{-1}\left(i_t^K\right))K_t + \widetilde{\tau}_M\left(1+i_t^D\right)\left(p_t^C C_t + p_t^V V_t\right). \end{aligned} \tag{111}$$

Finally, in the primary goods market, output supply equals intermediate demand by domestic firms (58) plus export demand:

$$A_t = c^q \frac{mc_t}{\left(1+\varsigma_t^A i_{t-1}^L\right) e_t p_t^{**A}} \left[Q_t + z_t F^D\right] + X_t^A. \tag{112}$$

9.2. GDP

Total imports N_t, is the sum of household and firm demand:

$$P_t^N N_t = (1-a_D)\, P_t^C C_t + (1-b_D)\, P_t^V V_t + P_t^N N_t^D. \tag{113}$$

Using (38), (45), and (113), we can express domestic output (111) as:

$$\begin{aligned} Q_t &= p_t^C C_t + p_t^V V_t + G_t + X_t^M - p_t^N\left(C_t^N + V_t^N - Q_t^{IN}\right) + Q_t^{ID} + p_t^N Q_t^{IN} \\ &= \left(p_t^C C_t + p_t^V V_t + G_t + X_t^M + p_t^A X_t^A - p_t^N N_t\right) - p_t^A X_t^A + Q_t^{ID} + p_t^N Q_t^{IN} \\ &= Y_t - p_t^A X_t^A + Q_t^{ID} + p_t^N Q_t^{IN}. \end{aligned} \tag{114}$$

where we defined intermediate output of domestic and imported origin and real GDP in terms of domestic goods as:

$$Q_t^{ID} = Q_t^{DA} + z_t C_{t+1}^B + \tau_u((\tau_u')^{-1}\left(i_t^K\right))K_t + \widetilde{\tau}_M\left(1+i_t^D\right)\left(p_t^C C_t + p_t^V V_t\right) \tag{115}$$

$$Q_t^{IN} = N_t^D. \tag{116}$$

$$Y_t = p_t^C C_t + p_t^V V_t + G_t + X_t^M + p_t^A X_t^A - p_t^N N_t. \tag{117}$$

9.3. The balance of payments

Using (26) in (27), the nominal aggregate household budget constraint (10) (where the Υ_t cancel out) becomes:

$$\begin{aligned} \left(M_t^{0,H} - M_{t-1}^{0,H}\right) + (D_t - D_{t-1}) &= \Pi_t + W_t h_t + \left[i_t^K u_t - \tau_u(u_t)\right] P_t K_t \\ &+ i_{t-1}^D D_{t-1} - \left[1+\widetilde{\tau}_M\left(\left(1+i_t^D\right)\right)\right]\left(P_t^C C_t + P_t^V V_t\right) - T_t. \end{aligned} \tag{118}$$

Here Π_t is the sum of profits from all four types of firms (domestic, primary, import, and manufactured export) as well as banks:

$$\begin{aligned}\Pi_t &= \Pi_t^D + \Pi_t^A + \Pi_t^N + \Pi_t^{MX} + \Pi_t^B \\ &= \left(P_t Q_t - W_t h_t - P_t i_t^K u_t K_t^D - S_t P_t^{**A} Q_t^{AD} - P_t^N N_t^D - i_t^L L_t^F\right) \\ &+ \left(S_t P_t^{**A}\left(X_t^A + Q_t^{AD}\right) - P_t Q_t^{DA} - P_t i_t^K u_t K_t^A\right) \\ &+ \left(P_t^N - S_t P_t^{**N}\right) N_t + \left(S_t P_t^{*MX} - P_t\right) X_t^M \\ &+ [i_{t-1}^L L_{t-1} + i_{t-1} B_{t-1}^{CB} - i_{t-1}^D D_{t-1} - \delta_t i_{t-1}^B S_{t-1} B_{t-1}^{*B}\left(1 - \gamma^{FX}\right) \\ &- \Delta S_t B_t^{*B}\left(1 - \gamma^{FX}\right) - C_t^B z_{t-1} P_{t-1}].\end{aligned} \tag{119}$$

Notice that we must subtract capital losses on bank foreign debt in order to obtain bank realized profits in period t. Consolidating (103), (118) and (119), taking into account (111)-(117) and the consolidated balance sheet constraint of banks and firms (by which loans to firms cancel out) yields the balance of payments equation:

$$\begin{aligned}&\left(R_t^{*CB} - R_{t-1}^{*CB}\right) - \left(B_t^{*G} - B_{t-1}^{*G}\right) - \left(B_t^{*B} - B_{t-1}^{*B}\right) = \\ &TB_t + i_{t-1}^{**} R_{t-1}^{*CB} - i_{t-1}^G B_{t-1}^{*G} - i_{t-1}^B B_{t-1}^{*B}.\end{aligned} \tag{120}$$

where we defined the trade balance as:

$$TB_t \equiv p_t^{*MX} X_t^M + p_t^{**A} X_t^A - N_t.$$

Below we also use an equation that consolidates the balance of payments equation and the consolidated public sector equation (104):

$$R_t^{*CB} - B_t^{*B} = \left(1 + i_{t-1}^{**}\right) R_{t-1}^{*CB} - \left(1 + i_{t-1}^B\right) B_{t-1}^{*B} + TB_t - GD_t. \tag{121}$$

10. Monetary Policy

We have endeavored to include banks and the central bank with some detail in order to be able to consider alternative monetary (including exchange rate) policies within a unified framework, and, in particular, to model an IT-MEF regime. The latter is the regime we use for the numerical solution in a forthcoming paper. In this regime, the Central Bank, through its regular interventions in the interbank and foreign exchange markets, is able to aim for the achievement of an operational target for the interbank interest rate i_t, and to simultaneously "lean against the wind" of real currency depreciations e_t/e_{t-1} (or appreciations) through two corresponding simple policy feedback rules.

More generally, we define alternative monetary policies according to the nominal anchor that prevails and how the operational targets and feedback rules are defined, and consider two broad classes of monetary policies: crawling pegs, in which the nominal anchor is the nominal exchange rate, and inflation targeting, in which the nominal anchor is the target rate of inflation. In the case of crawling pegs, the Central Bank mainly intervenes in the foreign exchange market, aiming to achieve a certain rate of nominal depreciation of the domestic currency. In the case of inflation targeting, the Central Bank mainly intervenes in the interbank market, aiming to achieve a certain operational target for the (short run) nominal interest rate that it considers appropriate for reaching a target inflation rate. There are consequently two *pure* (or *corner*) monetary policy regimes. In the case of a Crawling Peg with a Pure Interest Rate Float (CP-PIF) policy, the Central Bank does not actively intervene at all in the interbank market. By this we mean that the Central Bank's international reserves grow at the economy's trend growth rate. A particular case of this regime is a fixed exchange rate.[12] The other pure regime is Inflation Targeting with a Pure Exchange Rate Float (IT-PEF). In this regime the Central Bank does not actively intervene at all in the foreign exchange market, by which we mean that its real peso bond liabilities grow with the economy's trend rate of growth.

The latter is the case that draws the greatest attention in the literature, due perhaps to the much higher degree of exchange rate flexibility that exists in developed countries and their high and increasing use of an inflation targeting monetary policy. In a few developed countries, such as the U.S.A. and Japan, foreign exchange market interventions are sporadic and mainly have a signaling purpose. However, this is not the typical situation, and in most developed countries foreign exchange market intervention with the purpose of influencing the nominal exchange rate is quite common (see Bofinger and Wollmerhaüser (2001) and Wollmerhaüser (2003)). In developing countries daily foreign exchange market intervention is even more frequent and is often the most important policy action that the Central Bank exerts.

Since our model is mainly intended to be used in developing countries, we construct it so as to allow for a wide range of alternative monetary policies. In the following we consider the two "pure" (atypical) extremes just mentioned, but mainly develop a benchmark case of a "mixed" monetary policy: inflation targeting with a managed exchange rate float (IT-MEF), in which the Central Bank pursues an inflation rate target through two simple feedback rules; one for the operational target for the ("money market") interest rate and another that defines its intervention policy in the foreign exchange market.

Below we consider these alternative monetary policies more explicitly. The focus is more on obtaining a consistent framework that can deal with the actual complexities of monetary policies in developing countries than on the proposal or analysis of a particular "mixed" monetary policy with two parallel policy instruments.

10.1. Pure Exchange rate Crawl (PEC) regimes

We define a pure exchange rate crawl regime as one where the Central Bank abstains from actively intervening in the money market. Hence, it maintains its real liabilities in domestic currency bonds growing along with the economy's trend growth:

$$\frac{B_t^{CB}}{P_t} = b_0^{CB} z_t, \qquad \forall t.$$

Also, the Central Bank pegs the nominal exchange rate to the foreign currency by intervening in the foreign exchange market so as to ensure that the rate of nominal depreciation follows a predetermined target path $\{\delta_t^T\}$ such that $S_t/S_{t-1} = \delta_t^T$, for all t. We restrict attention to paths that converge to a constant δ^T in

[12] In particular, a Currency Board is an extreme version of a fixed exchange regime where the level of the nominal exchange rate is meant to be fixed "for all times".

a finite time. This implies that the Central Bank purchases any excess supply or satisfies any excess demand of foreign exchange that the private sector may have at the nominal exchange rate $S_t = S_{t-1}\delta_t^T$. We could formalize this as an infinitely fast feedback rule in which the Central Bank counteracts (excessive) nominal appreciations (depreciations) by purchasing (selling) international reserves (thus "leaning against the wind"). In the case considered here the Central Bank counteracts any deviation whatsoever of the rate of nominal depreciation from its target path. Hence, the stock of international reserves is endogenous and the following equation must be included in the system:

$$\delta_t = \delta_t^T, \qquad \forall t. \tag{122}$$

In the particular case of a fixed crawling peg policy the nominal rate of depreciation is kept at a constant level δ^T, and in the particular case of a fixed exchange rate policy, that constant level is unity.[13]

10.2. Inflation Targeting (IT) regimes

Under Inflation Targeting there are various possibilities for monetary policy feedback rules that can define the Central Bank's operational target for the nominal (domestic currency) interest rate i_t. A fairly general one is one where the Central Bank simultaneously responds to deviations of the gross domestic inflation rate from a target path $\{\pi_t^T\}$, to deviations of the consumption inflation rate (46) from a target path $\{\pi_t^{CT}\}$, to deviations of the trend adjusted output level from a target path $\{(Y_t/z_t)_t^T\}$, and possibly also to deviations of the RER from a target path $\{e_t^T\}$, as suggested by Smets and Wouters (2002) and ALLV(2005a). All these target paths, if they are time varying, are assumed to converge to a constant in finite time, and the target paths for detrended output and the RER must converge to their non-stochastic steady state levels. We also introduce history dependence for the nominal interest rate through the presence of the lagged interest rate. The simple interest rate feedback rule is thus the following:

$$1 + i_t = \left(\Xi^{TR}\right)^{1-h_0} (1 + i_{t-1})^{h_0} \left(\frac{\pi_t}{\pi_t^T}\right)^{h_1} \left(\frac{\pi_t^C}{\pi_t^{CT}}\right)^{h_2} \left(\frac{Y_t/z_t}{(Y_t/z_t)^T}\right)^{h_3} \left(\frac{e_t}{e_t^T}\right)^{h_4}, \tag{123}$$
$$h_0 \geq 0, h_1 > 0, h_2 > 0, h_3 \geq 0, h_4 \geq 0, h_1 + h_2 > 1, h_1 h_2 = 0.$$

where

$$\Xi^{TR} \equiv \frac{\pi^T}{\pi}\left\{1 + \frac{\pi}{\pi^{**N}}\left[(1 + i^{**})\,\varphi_B\left(\frac{B^{*B}e}{zP^{**N}}\right) - 1\right]\right\}. \tag{124}$$

Several comments are in order. 1) The (rather awkward) multiplicative term (124) in the feedback rule is designed so as to obtain a consistent non-stochastic steady state for the model, where the (steady state) inflation target is achieved. It does so by making appropriate use of the model's "uncovered interest parity condition" (96), which is here derived from the banks' optimization problem. We deal with the steady state at length in section 13 below. 2) We assume that only one of the inflation targets is used ($h_1 h_2 = 0$) and normally assume that the feedback rule has the "Taylor property" ($h_1 > 1$ or $h_2 > 1$) whereby the Central

[13] In the even more particular case of a Currency Board, it is unity "for all times".

Bank responds to excess expected inflation (either domestic or CPI) by increasing the expected real interest rate (and not merely the nominal interest rate). 3) We normally assume that the interest rate smoothing coefficient h_0 is not greater than one, but do not discard the possibility of having "superinertial" policy rules for the nominal interest rate ($h_0 > 1$) (see Woodford (2003), chapter 2). 4) We allow for the possibility that the interest rate feedback rule respond to the RER, since this relative price is particularly important in most developing countries in determining not only inflation but also their (short run) competitiveness in foreign markets. 5) A possible variant is to have the interest rate feedback rule respond to wage inflation. In that case π_t could be replaced by π_t^w in (123). 6) Another variant for the feedback rule has a forward looking reaction function, replacing the deviation of inflation from target by the expected deviation for next period(s) $E_t\left(\widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^T\right)$ (and similarly for the CPI or wage inflation rates). 7) In a quarterly model, the target rate of inflation would typically be a year on year rate π_t^A whereas π_t would be a quarter on quarter rate. Hence, we would have to add the equation that relates the two, the log-linear version of which is:

$$\widehat{\pi}_t^A = \widehat{\pi}_t + \widehat{\pi}_{t-1} + \widehat{\pi}_{t-2} + \widehat{\pi}_{t-3}$$

(and similarly for the CPI or wage inflation target rates)

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

We define an Inflation Targeting under a Pure Exchange Rate Float (IT-PEF) regime, as one where, in addition to following a simple interest rate feedback rule, the Central Bank abstains from actively intervening in the foreign exchange market. By this we mean that it maintains the real value of its international reserves growing along with the economy's trend growth:

$$\frac{R_t^{*CB}}{P_t^{**N}} = r_0 z_t, \qquad \forall t.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

Alternatively, we define Inflation Targeting under a Managed Exchange rate Float (IT-MEF) regime as one in which, in addition to following a simple interest rate feedback rule, the Central Bank actively intervenes in the foreign exchange market. We assume that aside from its operational target for the nominal interest rate, the Central Bank also has a long run operational target for the level of international reserves. The following is one possible feedback rule for the international reserves, in which the Central Bank tends to "lean against" real appreciations or depreciations (last multiplicative term), has a preference for smoothing the variations in the level of international reserves, and also has a long run target (γ^T) for the fraction of total financial system liabilities that are backed by Central Bank international reserves:

$$\frac{R_t^{*CB}}{P_t^{**N} z_t} = \left(\Xi^{FX}\right)^{1-k_0} \left(\frac{R_{t-1}^{*CB}}{P_{t-1}^{**N} z_{t-1}}\right)^{k_0} \left(\frac{e_t}{e_{t-1}}\right)^{-k_1}$$

$$k_0 \in (0,1), k_1 > 0.$$

where

$$\Xi^{FX} \equiv \gamma^{T} \frac{M^{0,H} + D + \left(1 - \gamma^{FX}\right) eB^{*B}/P^{**N}}{zPe} \tag{125}$$

The first multiplicative term (125) is designed so as to have a consistent steady state in which the reserves target is satisfied. Notice that under this policy feedback rule the Central Bank does not aim at any specific level of the nominal or real exchange rate. However, it does have a policy of "leaning against the wind" by increasing the purchase of reserves whenever there is real peso appreciation ($e_t < e_{t-1}$). The nominal anchor is still clearly the target inflation rate, as when there is a pure float.

11. Putting (most of) the non-linear system together

In this section we put together the non-linear equations thus far encountered that are common to all the possible monetary regimes. For clarity, we gather the equations in a few categories and give each a distinctive name that characterizes it. In these equations we have used (71) to eliminate p_t^A, (92) and (99) to eliminate i_t^B and i_t^G, and (116) and (59) to eliminate Q_t^{IN} and N_t^D.

Non-policy dynamic equations:

Consumption:

$$\frac{z_t^C}{C_t - \xi C_{t-1}} - \beta \xi E_t \left(\frac{z_{t+1}^C}{C_{t+1} - \xi C_t} \right) = \lambda_t \widetilde{\varphi}_M \left(1 + i_t^D\right)$$

Investment:

$$\zeta_t z_t^V \varphi_V \left(\frac{V_t}{V_{t-1}} \right) + \beta E_t \left\{ \zeta_{t+1} z_{t+1}^V \tau_V' \left(\frac{V_{t+1}}{V_t} \right) \left(\frac{V_{t+1}}{V_t} \right)^2 \right\} = \lambda_t \widetilde{\varphi}_M \left(1 + i_t^D\right)$$

Marginal utility of installed physical capital:

$$\zeta_t = \beta E_t \left\{ \left(1 - \delta^K\right) \zeta_{t+1} + \lambda_{t+1} \Gamma^K \left(i_{t+1}^K\right) \right\}$$

Marginal utility of real income:

$$\lambda_t = \beta \left(1 + i_t^D\right) E_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right)$$

Physical capital accumulation:

$$K_{t+1} = \left(1 - \delta^K\right) K_t + z_t^V V_t \left[1 - \tau_V \left(\frac{V_t}{V_{t-1}} \right) \right],$$

Wage inflation Phillips equations:

$$\begin{aligned} 0 \quad = \quad & E_t \sum_{j=0}^{\infty} \left(\beta \alpha_W\right)^j \lambda_{t+j} w_{t+j} h_{t+j} \left(\pi_{t+j}^w\right)^{\psi} \left\{ \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right) \right. \\ & \left. - \frac{\psi}{\psi - 1} \frac{\eta_H z_{t+j}^H h_{t+j}^{\chi}}{\lambda_{t+j} w_{t+j}} \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w} \right)^{-\psi \chi} \right\} \end{aligned}$$

$$\left(\pi_t^w\right)^{1-\theta} = \alpha_W \left(\pi_{t-1}^w\right)^{1-\theta} + (1-\alpha_W)\left(\widetilde{w}_t \pi_t^w\right)^{1-\theta}$$

Domestic inflation Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha)^j \,\overline{\Lambda}_{t+j} Q_{t+j} \left(\pi_{t+j}\right)^{\theta} \left\{ \frac{\widetilde{p}_t \pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1} mc_{t+j} \right\},$$

$$\pi_t^{1-\theta} = \alpha \pi_{t-1}^{1-\theta} + (1-\alpha)\left(\widetilde{p}_t \pi_t\right)^{1-\theta}.$$

Imported goods inflation Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_N)^j \,\overline{\Lambda}_{t+j} N_{t+j} (\pi_{t+j}^N)^{\theta_N} \left\{ \frac{\widetilde{p}_t^N \pi_t^N}{\pi_{t+j}^N} - \frac{\theta_N}{\theta_N - 1} \frac{e_{t+j}}{p_{t+j}^N} \right\}.$$

$$\left(\pi_t^N\right)^{1-\theta_N} = \alpha_N \left(\pi_{t-1}^N\right)^{1-\theta_N} + (1-\alpha_N)\left(\widetilde{p}_t^N \pi_t^N\right)^{1-\theta_N}.$$

Manufactured export goods inflation Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_X)^j \,\overline{\Lambda}_{t+j} X_{t+j} (\pi_{t+j}^{*X})^{\theta^*} \left\{ \frac{\widetilde{p}_t^{*X} \pi_t^{*X}}{\pi_{t+j}^{*X}} - \frac{\theta^*}{\theta^* - 1} \frac{1}{e_{t+j} p_{t+j}^{*X}} \right\}.$$

$$\left(\pi_t^{*X}\right)^{1-\theta^*} = \alpha_X \left(\pi_{t-1}^{*X}\right)^{1-\theta^*} + (1-\alpha_X)\left(\widetilde{p}_t^{*X} \pi_t^{*X}\right)^{1-\theta^*}.$$

Identities:

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\frac{p_t^N}{p_{t-1}^N} = \frac{\pi_t^N}{\pi_t}$$

$$\frac{p_t^{*MX}}{p_{t-1}^{*MX}} = \frac{\pi_t^{*MX}}{\pi_t^{**N}}$$

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi_t^{**N}}{\pi_t}$$

Fiscal:

$$\frac{e_t B_t^{*G}}{P_t^{**N}} = (1+i_{t-1}^{**})\left[1 + \phi_{t-1}^{**G} + p_G\left(\left(\frac{B_{t-1}^{*G}}{z_{t-1} P_{t-1}^{**N}} - \frac{R_{t-1}^{*CB}}{z_{t-1} P_{t-1}^{**N}}\right) e_{t-1}\right)\right] \frac{e_t B_{t-1}^{*G}}{\pi_t^{**N} P_{t-1}^{**N}} - \frac{QF_t}{P_t} - \frac{GD_t}{P_t}.$$

Central Bank quasi-fiscal surplus:

$$\frac{QF_t}{P_t} \equiv \left(1 + i_{t-1}^{**} - \frac{1}{\delta_t}\right) \frac{e_t R_{t-1}^{*CB}}{\pi_t^{**N} P_{t-1}^{**N}} - i_{t-1} \frac{B_{t-1}^{CB}}{P_t}$$

Government domestic surplus:

$$\frac{GD_t}{P_t} \equiv \frac{T_t}{P_t} - G_t + \frac{L_t^G - \left(1 + i_{t-1}^L\right) L_{t-1}^G}{P_t}$$

Balance of Payments:

$$\frac{R_t^{*CB}}{P_t^{**N}} - \frac{B_t^{*B}}{P_t^{**N}} - \frac{B_t^{*G}}{P_t^{**N}} = TB_t + \left(1 + i_{t-1}^{**}\right) \frac{R_{t-1}^{*CB}}{\pi_t^{**N} P_{t-1}^{**N}}$$
$$-(1 + i_{t-1}^{**}) \left[1 + \phi_{t-1}^{**B} + p_B \left(\frac{B_{t-1}^{*B} e_{t-1}}{z_{t-1} P_{t-1}^{**N}}\right)\right] \frac{B_{t-1}^{*B}}{\pi_t^{**N} P_{t-1}^{**N}}$$
$$-(1 + i_{t-1}^{**}) \left[1 + \phi_{t-1}^{**G} + p_G \left(\left(\frac{B_{t-1}^{*G}}{z_{t-1} P_{t-1}^{**N}} - \frac{R_{t-1}^{*CB}}{z_{t-1} P_{t-1}^{**N}}\right) e_{t-1}\right)\right] \frac{B_{t-1}^{*G}}{\pi_t^{**N} P_{t-1}^{**N}}.$$

Trade balance:

$$TB_t \equiv p_t^{*MX} X_t^M + p_t^{**A} X_t^A - N_t$$

Bank arbitrage:

$$1 + i_t = 1 + \left[\beta^B E_t \delta_{t+1} + \left(1 - \beta^B\right) \delta_t\right] \left[(1 + i_t^{**}) \varphi_B \left(\frac{S_t B_t^{*B}}{P_t z_t}\right) - 1\right]$$

Loan market clearing:

$$\frac{L_t}{P_t} = E_t\{f_L \left(1 + i_t^L\right) mc_{t+1} \left[Q_{t+1} + z_{t+1} F^D\right]\} + \frac{L_t^G}{P_t}.$$

Real marginal cost:

$$mc_t = \frac{1}{\kappa \epsilon_t} f_{MC} \left(1 + i_{t-1}^L\right) \left(i_t^K\right)^{a^q} \left(\frac{w_t}{z_t}\right)^{b^q} \left(p_t^A\right)^{c^q} \left(p_t^N\right)^{1-a^q-b^q-c^q}.$$

Import demand:

$$p_t^N N_t = (1 - a_D) p_t^C C_t + (1 - b_D) p_t^V V_t + \frac{(1 - a^q - b^q - c^q) mc_t}{1 + \varsigma_t^N i_{t-1}^L} \left[Q_t + z_t F^D\right].$$

Physical capital rental market clearing:

$$(\tau_u')^{-1} \left(i_t^K\right) K_t = a^q \frac{mc_t}{\left(1 + \varsigma_t^W i_{t-1}^L\right) i_t^K} \left[Q_t + z_t F^D\right] + \beta_A \frac{e_t p^{**A}}{i_t^K} A_t.$$

Labor market clearing:

$$h_t = b^q \frac{mc_t}{\left(1 + \varsigma_t^W i_{t-1}^L\right) w_t} \left[Q_t + z_t F^D\right]$$

Domestic goods market clearing:

$$Q_t = Y_t - e_t p^{**A} X_t^A + Q_t^{ID} + \frac{(1 - a^q - b^q - c^q) mc_t}{1 + \varsigma_t^N i_{t-1}^L} \left[Q_t + z_t F^D\right].$$

Primary exports:

$$X_t^A = A_t - c^q \frac{mc_t}{\left(1 + \varsigma_t^A i_{t-1}^L\right) e_t p_t^{**A}} \left[Q_t + z_t F^D\right].$$

Static equations:

Deposit market clearing:

$$\frac{D_t}{P_t} = \frac{z_t}{a^B}\left\{a_L^B\left[(1-\gamma_t^B-\gamma_t^R)i_t - i_t^D\right] + a_0^B\left(i_t^L - i_t\right)\right\}$$

Interbank cum Central Bank bond market clearing:

$$\frac{B_t^{CB}}{P_t} = (1-\gamma_t^B-\gamma_t^R)\frac{D_t}{P_t} + \left(1-\gamma^{FX}\right)e_t\frac{B_t^{*B}}{P_t^{**N}} - \frac{L_t}{P_t}.$$

Cash market clearing:

$$\frac{M_t^0}{P_t} = \mathcal{L}\left(1+i_t^D\right)\left[p_t^C C_t + p_t^V V_t\right] + \gamma_t^B\frac{D_t}{P_t}$$

Real GDP

$$Y_t = p_t^C C_t + p_t^V V_t + G_t + X_t^M + e_t p^{**A} X_t^A - p_t^N N_t.$$

Intermediate demand for domestic goods:

$$Q_t^{ID} = \alpha_A e_t p^{**A} A_t + z_t C_{t+1}^B + \widetilde{\tau}_M\left(1+i_t^D\right)\left(p_t^C C_t + p_t^V V_t\right) + \tau_u((\tau_u')^{-1}\left(i_t^K\right))K_t$$

Loan supply:

$$\frac{L_t}{P_t} = \frac{z_t}{a^B}\left\{a_D^B\left(i_t^L - i_t\right) + a_0^B\left[(1-\gamma_t^B-\gamma_t^R)i_t - i_t^D\right]\right\}$$

Central Bank balance sheet:

$$\frac{B_t^{CB}}{P_t} = e_t\frac{R_t^{*CB}}{P_t^{**N}} - \frac{M_t^0}{P_t} - \gamma_t^R\frac{D_t}{P_t}.$$

Primary goods supply:

$$A_t = \left(\kappa_A\frac{\left(e_t p_t^{**A}\right)^{\alpha_A+\beta_A}}{\left(i_t^K\right)^{\beta_A}}\right)^{\frac{1}{1-\alpha_A-\beta_A}}$$

Manufactured exports demand:

$$X_t^M = Y_t^{**}\left(p_t^{*MX} p_t^{**X}\right)^{-\theta^{**}}$$

Consumption inflation rate

$$\pi_t^C = \left[\frac{a_D}{a_D + a_N\left(p_{t-1}^N\right)^{1-\theta_C}}\left(\pi_t\right)^{1-\theta_C} + \left(1 - \frac{a_D}{a_D + a_N\left(p_{t-1}^N\right)^{1-\theta_C}}\right)\left(\pi_t^N\right)^{1-\theta_C}\right]^{\frac{1}{1-\theta_C}},$$

Bank real cost:

$$C_{t+1}^B = \frac{1}{2}\left[a_L^B\left(\frac{L_t}{z_t P_t}\right)^2 + a_D^B\left(\frac{D_t}{z_t P_t}\right)^2 - 2a_0^B\left(\frac{L_t}{z_t P_t}\right)\left(\frac{D_t}{z_t P_t}\right)\right].$$

Consumption relative price:

$$p_t^C = \left[a_D + (1 - a_D) \left(p_t^N \right)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}} .$$

Investment relative price:

$$p_t^V = \left[b_D + (1 - b_D) \left(p_t^N \right)^{1-\theta_V} \right]^{\frac{1}{1-\theta_V}} .$$

So far we have 39 equations to determine the following 41 endogenous variables:
4 rates of return: i_t^K, i_t, i_t^L, i_t^D,
5 rates of inflation: π_t^w, π_t, π_t^N, π_t^{*X}, δ_t,
11 relative prices: p_t^N, p_t^{*X}, w_t, e_t, $\widetilde{w}_t$, $\widetilde{p}_t$, $\widetilde{p}_t^N$, $\widetilde{p}_t^{*X}$, mc_t, p_t^C, p_t^V
11 flows: C_t, V_t, h_t, N_t, X_t^M, X_t^A, A_t, Q_t, Q_t^{ID}, Y_t, C_t^B
8 stocks: K_t, M_t^0/P_t, R_t^{*CB}/P_t^{**N}, B_t^{CB}/P_t, D_t/P_t, L_t/P_t, B_t^{*B}/P_t^{**N}, B_t^{*G}/P_t^{**N},
2 Lagrange multipliers: λ_t, ζ_t.

Hence, we have room for the two monetary policy equations that define the alternative monetary regimes. Instead of listing them now again (see the section on the Central Bank), we do so in the next section, where we put the model in terms of stationary variables. Some additional system equations will be appended when we specify the assumptions on the relation between the SOE's and the LRW's growth rates.

12. The non-linear equations in stationary format

In order to have a well defined steady state we need to express the system's equations in terms of stationary variables. The only source of growth in this model is technological progress, so we use lower case letters to express upper case letter variables when deflated by the permanent technology shock in the production of domestic goods z_t (or z_t^{**} in the case of the LRW's variables), and add a superscript $^\circ$ to the Lagrange multipliers to denote that they are inflated by the same factor. We use this same superscript to denote the LRW's permanent technology shock relative to the SOE's (z_t°) (which is lower case to begin with).

$$\begin{aligned}
\overline{w}_t &= \frac{w_t}{z_t} = \frac{W_t}{P_t z_t}, \quad c_t = \frac{C_t}{z_t}, \quad v_t = \frac{V_t}{z_t}, \quad q_t = \frac{Q_t}{z_t}, \quad y_t = \frac{Y_t}{z_t}, \\
q_t^{ID} &= \frac{Q_t^{ID}}{z_t}, \quad k_{t+1} = \frac{K_{t+1}}{z_t}, \quad n_t = \frac{N_t}{z_t}, \quad x_t^M = \frac{X_t^M}{z_t}, \quad x_t^A = \frac{X_t^A}{z_t}, \\
a_t &= \frac{A_t}{z_t}, \quad g_t = \frac{G_t}{z_t}, \quad m_t^0 = \frac{M_t^0}{P_t z_t}, \quad d_t = \frac{D_t}{P_t z_t}, \quad b_t^{CB} = \frac{B_t^{CB}}{P_t z_t}, \\
r_t^{*CB} &= \frac{R_t^{*CB}}{P_t^{**N} z_t}, \quad b_t^{*B} = \frac{B_t^{*B}}{P_t^{**N} z_t}, \quad b_t^{*G} = \frac{B_t^{*G}}{P_t^{**N} z_t}, \quad t_t = \frac{T_t}{P_t z_t}, \\
\ell_t &= \frac{L_t}{P_t z_t}, \quad \ell_t^G = \frac{L_t^G}{P_t z_t}, \quad y_t^{**} = \frac{Y_t^{**}}{z_t^{**}} \quad \lambda_t^\circ = \lambda_t z_t, \quad \zeta_t^\circ = \zeta_t z_t, \\
\overline{\Lambda}_t^\circ &= \overline{\Lambda}_t z_t, \quad z_t^\circ = \frac{z_t^{**}}{z_t} \quad qf_t = \frac{QF_t}{z_t P_t} \quad gd_t = \frac{GD_t}{P_t} \quad tb_t = \frac{TB_t}{P_t^{**N} z_t}.
\end{aligned}$$

We also define the growth rate of the permanent technology shock:

$$\mu_t^z \equiv \frac{z_t}{z_{t-1}}. \qquad (127)$$

Hence, we rewrite the equations of the nonlinear system in stationary format as:

Non-policy dynamic equations:

Consumption:

$$\mu_t^z\left(\frac{z_t^C}{c_t\mu_t^z-\xi c_{t-1}}\right)-\beta\xi E_t\left(\frac{z_{t+1}^C}{c_{t+1}\mu_{t+1}^z-\xi c_t}\right)=\lambda_t^\circ\widetilde{\varphi}_M\left(1+i_t^D\right)$$

Investment:

$$\zeta_t^\circ z_t^V\varphi_V\left(\frac{v_t}{v_{t-1}}\mu_t^z\right)+\beta E_t\left\{\frac{\zeta_{t+1}^\circ z_{t+1}^V}{\mu_{t+1}^z}\tau_V'\left(\frac{v_{t+1}}{v_t}\mu_{t+1}^z\right)\left(\frac{v_{t+1}}{v_t}\mu_{t+1}^z\right)^2\right\}$$
$$=\lambda_t^\circ\widetilde{\varphi}_M\left(1+i_t^D\right)$$

Marginal utility of installed physical capital:

$$\zeta_t^\circ=\beta E_t\left\{\left(1-\delta^K\right)\left(\frac{\zeta_{t+1}^\circ}{\mu_{t+1}^z}\right)+\left(\frac{\lambda_{t+1}^\circ}{\mu_{t+1}^z}\right)\Gamma^K\left(i_{t+1}^K\right)\right\}$$

Marginal utility of real income:

$$\lambda_t^\circ=\beta\left(1+i_t^D\right)E_t\left(\frac{\lambda_{t+1}^\circ}{\mu_{t+1}^z\pi_{t+1}}\right)$$

Physical capital accumulation:

$$k_{t+1}=\left(1-\delta^K\right)\frac{k_t}{\mu_t^z}+z_t^V v_t\left[1-\tau_V\left(\frac{v_t}{v_{t-1}}\mu_t^z\right)\right],$$

Wage inflation Phillips equations:

$$0=E_t\sum_{j=0}^{\infty}\left(\beta\alpha_W\right)^j\lambda_{t+j}^\circ\overline{w}_{t+j}h_{t+j}\left(\pi_{t+j}^w\right)^\psi\left\{\left(\frac{\widetilde{w}_t\pi_t^w}{\pi_{t+j}^w}\right)\right.$$
$$\left.-\frac{\psi}{\psi-1}\frac{\eta_H z_{t+j}^H h_{t+j}^\chi}{\lambda_{t+j}^\circ\overline{w}_{t+j}}\left(\frac{\widetilde{w}_t\pi_t^w}{\pi_{t+j}^w}\right)^{-\psi\chi}\right\}$$

$$\left(\pi_t^w\right)^{1-\theta}=\alpha_W\left(\pi_{t-1}^w\right)^{1-\theta}+\left(1-\alpha_W\right)\left(\widetilde{w}_t\pi_t^w\right)^{1-\theta}$$

Domestic inflation Phillips equations:

$$0=E_t\sum_{j=0}^{\infty}\left(\beta\alpha\right)^j\overline{\Lambda}_{t+j}^\circ q_{t+j}\left(\pi_{t+j}\right)^\theta\left\{\frac{\widetilde{p}_t\pi_t}{\pi_{t+j}}-\frac{\theta}{\theta-1}mc_{t+j}\right\},$$
$$\pi_t^{1-\theta}=\alpha\pi_{t-1}^{1-\theta}+\left(1-\alpha\right)\left(\widetilde{p}_t\pi_t\right)^{1-\theta}.$$

Imported goods inflation Phillips equations:

$$0=E_t\sum_{j=0}^{\infty}\left(\beta\alpha_N\right)^j\overline{\Lambda}_{t+j}^\circ n_{t+j}(\pi_{t+j}^N)^{\theta_N}\left\{\frac{\widetilde{p}_t^N\pi_t^N}{\pi_{t+j}^N}-\frac{\theta_N}{\theta_N-1}\frac{e_{t+j}}{p_{t+j}^N}\right\}.$$

$$\left(\pi_t^N\right)^{1-\theta_N}=\alpha_N\left(\pi_{t-1}^N\right)^{1-\theta_N}+\left(1-\alpha_N\right)\left(\widetilde{p}_t^N\pi_t^N\right)^{1-\theta_N}.$$

Manufactured export goods inflation Phillips equations:

$$0 = E_t \sum_{j=0}^{\infty} (\beta\alpha_X)^j \, \overline{\Lambda}^{\circ}_{t+j} x^M_{t+j} (\pi^{*MX}_{t+j})^{\theta^*} \left\{ \frac{\widetilde{p}^{*MX}_t \pi^{*MX}_t}{\pi^{*MX}_{t+j}} - \frac{\theta^*}{\theta^* - 1} \frac{1}{e_{t+j} p^{*MX}_{t+j}} \right\}.$$

$$\left(\pi^{*MX}_t\right)^{1-\theta^*} = \alpha_X \left(\pi^{*MX}_{t-1}\right)^{1-\theta^*} + (1-\alpha_X) \left(\widetilde{p}^{*MX}_t \pi^{*MX}_t\right)^{1-\theta^*}.$$

Identities:

$$\frac{\overline{w}_t}{\overline{w}_{t-1}} = \frac{\pi^w_t}{\pi_t \mu^z_t}$$

$$\frac{p^N_t}{p^N_{t-1}} = \frac{\pi^N_t}{\pi_t}$$

$$\frac{p^{*MX}_t}{p^{*MX}_{t-1}} = \frac{\pi^{*MX}_t}{\pi^{**N}_t}$$

$$\frac{e_t}{e_{t-1}} = \frac{\delta_t \pi^{**N}_t}{\pi_t} \tag{129}$$

Fiscal:

$$b^{*G}_t = (1 + i^{**}_{t-1}) \left[1 + \phi^{**G}_{t-1} + p_G \left(\left(b^{*G}_{t-1} - r^{*CB}_{t-1} \right) e_{t-1} \right) \right] \frac{b^{*G}_{t-1}}{\mu^z_t \pi^{**N}_t} - \frac{1}{e_t} (qf_t + gd_t). \tag{130}$$

Central Bank quasi-fiscal surplus:

$$qf_t \equiv \left(1 + i^{**}_{t-1} - \frac{1}{\delta_t}\right) \frac{e_t r^{*CB}_{t-1}}{\pi^{**N}_t \mu^z_t} - i_{t-1} b^{CB}_{t-1}$$

Government domestic surplus:

$$gd_t = t_t - g_t + \ell^G_t - \frac{1 + i^L_{t-1}}{\mu^z_t \pi_t} \ell^G_{t-1}$$

Balance of Payments:

$$\begin{aligned} r^{*CB}_t - b^{*B}_t - b^{*G}_t = tb_t + \left(1 + i^{**}_{t-1}\right) \Bigg\{ & \frac{r^{*CB}_{t-1}}{\mu^z_t \pi^{**N}_t} \\ & - \left[1 + \phi^{**B}_{t-1} + p_B \left(b^{*B}_{t-1} e_{t-1} \right) \right] \frac{b^{*B}_{t-1}}{\mu^z_t \pi^{**N}_t} \\ & - \left[1 + \phi^{**G}_{t-1} + p_G \left(\left(b^{*G}_{t-1} - r^{*CB}_{t-1} \right) e_{t-1} \right) \right] \frac{b^{*G}_{t-1}}{\mu^z_t \pi^{**N}_t} \Bigg\}. \end{aligned} \tag{131}$$

Trade balance:

$$tb_t = p^{*MX}_t x^M_t + p^{**A}_t x^A_t - n_t$$

Bank arbitrage:

$$1 + i_t = 1 + \left[\beta^B E_t \delta_{t+1} + \left(1 - \beta^B\right) \delta_t\right] \left[(1 + i^{**}_t)\, \varphi_B \left(e_t b^{*B}_t\right) - 1\right] \tag{132}$$

Loan market clearing:

$$\ell_t = E_t \{ \mu^z_{t+1} f_L \left(1 + i^L_t\right) mc_{t+1} \left[q_{t+1} + F^D\right] \} + \ell^G_t$$

Real marginal cost:

$$mc_t = \frac{1}{\kappa\epsilon_t} f_{MC} \left(1 + i_{t-1}^L\right) \left(i_t^K\right)^{a^q} \overline{w}_t^{b^q} \left(e_t p_t^{**A}\right)^{c^q} \left(p_t^N\right)^{1-a^q-b^q-c^q}.$$

Import demand:

$$p_t^N n_t = (1 - a_D)\, p_t^C c_t + (1 - b_D)\, p_t^V v_t + \frac{(1 - a^q - b^q - c^q) mc_t}{1 + \varsigma_t^N i_{t-1}^L} \left[q_t + F^D\right]$$

Physical capital rental market clearing:

$$(\tau_u')^{-1} \left(i_t^K\right) \frac{k_t}{\mu_t^z} = a^q \frac{mc_t}{\left(1 + \varsigma_t^K i_{t-1}^L\right) i_t^K} \left[q_t + F^D\right] + \beta_A \frac{e_t p_t^{**A}}{i_t^K} a_t$$

Labor market clearing:

$$h_t = b^q \frac{mc_t}{\left(1 + \varsigma_t^W i_{t-1}^L\right) \overline{w}_t} \left[q_t + F^D\right]$$

Domestic goods market clearing:

$$q_t = y_t - e_t p_t^{**A} x_t^A + q_t^{ID} + \frac{(1 - a^q - b^q - c^q) mc_t}{1 + \varsigma_t^N i_{t-1}^L} \left[q_t + F^D\right]$$

Primary exports:

$$x_t^A = a_t - c^q \frac{mc_t}{\left(1 + \varsigma_t^A i_{t-1}^L\right) e_t p_t^{**A}} \left[q_t + F^D\right]$$

Non-policy static equations:

Deposit market clearing:

$$d_t = \frac{1}{a^B} \left\{ a_L^B \left[(1 - \gamma_t^B - \gamma_t^R) i_t - i_t^D\right] + a_0^B \left(i_t^L - i_t\right) \right\}$$

Interbank cum Central Bank bond market clearing:

$$b_t^{CB} = (1 - \gamma_t^B - \gamma_t^R) d_t + \left(1 - \gamma^{FX}\right) e_t b_t^{*B} - \ell_t$$

Cash market clearing:

$$m_t^0 = \mathcal{L} \left(1 + i_t^D\right) \left[p_t^C c_t + p_t^V v_t\right] + \gamma_t^B d_t.$$

Real GDP:

$$y_t = p_t^C c_t + p_t^V v_t + g_t + x_t^M + e_t p_t^{**A} x_t^A - p_t^N n_t.$$

Intermediate demand for domestic goods:

$$q_t^{ID} = \alpha_A e_t p_t^{**A} a_t + C_{t+1}^B + \widetilde{\tau}_M \left(1 + i_t^D\right) \left(p_t^C c_t + p_t^V v_t\right) + \tau_u \left((\tau_u')^{-1} \left(i_t^K\right)\right) \frac{k_t}{\mu_t^z}$$

Loan supply

$$\ell_t = \frac{1}{a^B} \left\{ a_D^B \left(i_t^L - i_t\right) + a_0^B \left[(1 - \gamma_t^B - \gamma_t^R) i_t - i_t^D\right] \right\}$$

Central Bank balance sheet:

$$b_t^{CB} = e_t r_t^{*CB} - m_t^0 - \gamma_t^R d_t$$

Primary goods supply:

$$a_t = \left(\kappa_A \frac{\left(e_t p_t^{**A}\right)^{\alpha_A + \beta_A}}{\left(i_t^K\right)^{\beta_A}} \right)^{\frac{1}{1-\alpha_A-\beta_A}}$$

Manufactured exports demand:

$$x_t^M = z_t^\circ y_t^{**} \left(p_t^{*MX} p_t^{**X}\right)^{-\theta^{**}} \tag{133}$$

Consumption inflation rate

$$\pi_t^C = \left[\frac{a_D}{a_D + a_N \left(p_{t-1}^N\right)^{1-\theta_C}} \left(\pi_t\right)^{1-\theta_C} + \left(1 - \frac{a_D}{a_D + a_N \left(p_{t-1}^N\right)^{1-\theta_C}}\right) \left(\pi_t^N\right)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}$$

Bank real cost:

$$C_{t+1}^B = \frac{1}{2} \left[a_L^B \left(\ell_t\right)^2 + a_D^B \left(d_t\right)^2 - 2 a_0^B \ell_t d_t \right].$$

Consumption relative price:

$$p_t^C = \left[a_D + \left(1 - a_D\right) \left(p_t^N\right)^{1-\theta_C} \right]^{\frac{1}{1-\theta_C}}$$

Investment relative price:

$$p_t^V = \left[b_D + \left(1 - b_D\right) \left(p_t^N\right)^{1-\theta_V} \right]^{\frac{1}{1-\theta_V}}$$

Policy equations:

Pure Exchange rate Crawl regimes (PEC)

$$b_t^{CB} = b_0^{CB}$$

$$\delta_t = \delta_t^T.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$\begin{aligned} 1 + i_t &= \left(\Xi^{TR}\right)^{1-h_0} \left(1 + i_{t-1}\right)^{h_0} \left(\frac{\pi_t}{\pi_t^T}\right)^{h_1} \left(\frac{\pi_t^C}{\pi_t^{CT}}\right)^{h_2} \left(\frac{y_t}{y_t^T}\right)^{h_3} \left(\frac{e_t}{e_t^T}\right)^{h_4}, \\ \Xi^{TR} &\equiv \frac{\pi^T}{\pi} \left\{ 1 + \frac{\pi}{\pi^{**N}} \left[\left(1 + i^{**}\right) \varphi_B \left(b^{*B} e\right) - 1 \right] \right\} \end{aligned}$$

$$r_t^{*CB} = r_0 \qquad \forall t.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$1+i_t = \left(\Xi^{TR}\right)^{1-h_0}\left(1+i_{t-1}\right)^{h_0}\left(\frac{\pi_t}{\pi_t^T}\right)^{h_1}\left(\frac{\pi_t^C}{\pi_t^{CT}}\right)^{h_2}\left(\frac{y_t}{y_t^T}\right)^{h_3}\left(\frac{e_t}{e_t^T}\right)^{h_4},$$

$$r_t^{*CB} = \left(\Xi^{FX}\right)^{1-k_0}\left(r_{t-1}^{*CB}\right)^{k_0}\left(\frac{e_t}{e_{t-1}}\right)^{-k_1},$$

$$\Xi^{FX} \equiv \gamma^T\frac{m^{0,H}+d+\left(1-\gamma^{FX}\right)eb^{*B}}{e}.$$

We make two simplifications. First, notice that we can use (129) to eliminate the rate of nominal depreciation. In the case of the bank arbitrage condition we lead (129) and use the resulting expression in (132), obtaining:

$$1+i_t = 1+\left[\beta^B E_t\left(\frac{e_{t+1}}{e_t}\frac{\pi_{t+1}}{\pi_{t+1}^{**N}}\right)+\left(1-\beta^B\right)\frac{e_t}{e_{t-1}}\frac{\pi_t}{\pi_t^{**N}}\right]\left[\left(1+i_t^{**}\right)\varphi_B\left(e_t b_t^{*B}\right)-1\right]. \tag{134}$$

In the case of PEC regimes, the exchange rate policy equation (122) becomes:

$$\frac{e_t}{e_{t-1}}\frac{\pi_t}{\pi_t^{**N}} = \delta_t^T \qquad \forall t.$$

Second, notice that adding (130) and (131) term by term allows us to replace the balance of payments equation by the following simpler equation that combines the two and is simply (121) with the variables in stationary format:

$$r_t^{*CB}-b_t^{*B} = tb_t+\left(1+i_{t-1}^{**}\right)\left\{\frac{r_{t-1}^{*CB}}{\mu_t^z\pi_t^{**N}}-\left[1+\phi_{t-1}^{**B}+p_B\left(b_{t-1}^{*B}e_{t-1}\right)\right]\frac{b_{t-1}^{*B}}{\mu_t^z\pi_t^{**N}}\right\} \tag{135}$$

$$-\left(qf_t+gd_t\right)/e_t.$$

The fiscal equation (130) is decomposable from the rest (since b_t^{*G} does not show up in any of the remaining equations), so we may leave it out of the (core) system. Also, in any of the pure policies there is a variable we can convert to a constant. Obviously, many other variables may be substituted out of the system at the cost of having longer equations.

13. Analysis of the steady state

In this section we consider the non-stochastic steady states (SS) around which we make log-linear approximations to the dynamic systems that correspond to the alternative monetary policy regimes. We replace the stationary variables in the system by their non-stochastic steady state values (which we denote by the same variables without any time index), recalling that $\tau_V(\mu^z)=\tau_V'(\mu^z)=0$ (and hence $\varphi_V\left(\mu^V\right)=1$), and $\tau_u(u)=\tau_u(1)=0$ (and hence $\Gamma^K\left(i^K\right)=i^K$). For convenience (in the case of a PEC regime), we take the equations before eliminating δ_t. We also assume that the steady state growth rate of the SOE is equal to that of the LRW ($\mu^z=\mu^{z**}$, and hence $z^\circ=1$), and that the relative prices in the LRW (p^{**A}, p^{**X}) have a SS. We elaborate on the dynamics of μ_t^z in the next section. For simplicity, we normalize the following shocks to unity in the steady state: $z^C=z^V=z^H=z^M=\epsilon=1$. The model equations with the variables at their SS values are the following:

Non-policy equations:

Consumption:

$$\frac{1}{\lambda^{\circ}c}\frac{\mu^{z**}-\beta\xi}{\mu^{z**}-\xi}=\widetilde{\varphi}_{M}\left(1+i^{D}\right) \tag{136}$$

Investment:

$$\frac{\zeta^{\circ}}{\lambda^{\circ}}=\widetilde{\varphi}_{M}\left(1+i^{D}\right) \tag{137}$$

Marginal utility of installed physical capital:

$$\frac{\zeta^{\circ}}{\lambda^{\circ}}=\frac{1}{\mu^{z**}/\beta-\left(1-\delta^{K}\right)}i^{K} \tag{138}$$

Marginal utility of real income:

$$\mu^{z**}\pi=\beta\left(1+i^{D}\right) \tag{139}$$

Physical capital accumulation:

$$\frac{k}{\mu^{z**}v}=\frac{1}{\mu^{z**}-\left(1-\delta^{K}\right)}, \tag{140}$$

Wage inflation Phillips equations:

$$\widetilde{w}^{1+\psi\chi}=\frac{\psi}{\psi-1}\frac{\eta_{H}h^{\chi}}{\lambda^{\circ}\overline{w}} \tag{141}$$

$$1=\widetilde{w}$$

Domestic inflation Phillips equations:

$$\widetilde{p}=\frac{\theta}{\theta-1}mc, \tag{142}$$

$$1=\widetilde{p}.$$

Imported goods inflation Phillips equations:

$$\widetilde{p}^{N}=\frac{\theta_{N}}{\theta_{N}-1}\frac{e}{p^{N}} \tag{143}$$

$$1=\widetilde{p}^{N}$$

Manufactured export goods inflation Phillips equations:

$$\widetilde{p}^{*MX}=\frac{\theta^{*}}{\theta^{*}-1}\frac{1}{ep^{*MX}}. \tag{144}$$

$$1=\widetilde{p}^{*MX}$$

Identities:

$$1=\frac{\pi^{w}}{\pi\mu^{z}} \tag{145}$$

$$1 = \frac{\pi^N}{\pi} \tag{146}$$

$$1 = \frac{\pi^{*MX}}{\pi^{**N}} \tag{147}$$

$$1 = \frac{\delta\pi^{**N}}{\pi} \tag{148}$$

Balance of Payments cum Fiscal:

$$r^{*CB}\left(1 - \frac{1}{\mu^{z**}\pi}\right) - b^{*B}\left(1 - \frac{1+i^{**}}{\mu^{z**}\pi^{**N}}\left[1 + \phi_t^{**B} + p_B\left(eb^{*B}\right)\right]\right)$$
$$= tb - (qf + gd)/e.$$

Central Bank quasi-fiscal surplus:

$$qf \equiv \left(1 + i^{**} - \frac{1}{\delta_t}\right)\frac{er^{*CB}}{\pi^{**N}\mu^z} - ib^{CB}$$

Government domestic surplus:

$$gd = t - g + \ell^G\left(1 - \frac{1+i^L}{\mu^z\pi}\right)$$

Trade balance:

$$tb = p^{*MX}x^M + p^{**A}x^A - n$$

Bank arbitrage:

$$i = \delta\left[(1+i^{**})\,\varphi_B\left(eb^{*B}\right) - 1\right] \tag{149}$$

Loan market clearing:

$$\ell = \mu^{z**}f_L\left(1+i^L\right)mc\left[q + F^D\right] + \ell^G \tag{150}$$

Real marginal cost:

$$mc = \frac{1}{\kappa}f_{MC}\left(1+i^L\right)\left(i^K\right)^{a^q}\overline{w}^{b^q}\left(ep^{**A}\right)^{c^q}\left(p^N\right)^{1-a^q-b^q-c^q}.$$

Import demand:

$$p^N n = (1-a_D)\,p^C c + (1-b_D)\,p^V v + \frac{(1-a^q-b^q-c^q)mc}{1+\varsigma^N i^L}\left[q + F^D\right]$$

Physical capital rental market clearing:

$$\frac{ki^K}{\mu^{z**}} = \frac{a^q mc\left[q+F^D\right]}{1+\varsigma^K i^L} + \beta_A ep^{**A}a$$

Labor market clearing:

$$h\overline{w}\left[1+\varsigma^W i^L\right] = b^q mc\left[q + F^D\right]$$

Domestic goods market clearing:

$$q = y - ep^{**A}x^A + q^{ID} + \frac{(1-a^q-b^q-c^q)mc}{1+\varsigma^N i^L}\left[q+F^D\right]$$

Primary exports:

$$a = c^q \frac{mc}{(1+\varsigma^A i^L)\, ep^{**A}}\left[q+F^D\right] + x^A \tag{151}$$

Deposit market clearing:

$$d = \frac{1}{a^B}\left\{a_L^B\left[(1-\gamma^B-\gamma^R)i - i^D\right] + a_0^B\left(i^L - i\right)\right\} \tag{152}$$

Interbank cum Central Bank bond market clearing:

$$b^{CB} = (1-\gamma^B-\gamma^R)d + \left(1-\gamma^{FX}\right)eb^{*B} - \ell.$$

Cash market clearing:

$$m^0 = \mathcal{L}\left(1+i^D\right)\left[p^C c + p^V v\right] + \gamma^B d.$$

Real GDP:

$$y = p^C c + p^V v + g + x^M + ep^{**A}x^A - p^N n.$$

Intermediate demand for domestic goods:

$$q^{ID} = \alpha_A ep^{**A}a + C^B + \widetilde{\tau}_M\left(1+i^D\right)\left(p^C c + p^V v\right)$$

Loan supply

$$\ell = \frac{1}{a^B}\left\{a_D^B\left[i^L - i\right] + a_0^B\left[(1-\gamma^B-\gamma^R)i - i^D\right]\right\} \tag{153}$$

Central Bank balance sheet:

$$b^{CB} = er^{*CB} - m^0 - \gamma^R d$$

Primary good supply:

$$a = \left(\kappa_A \frac{\left(ep^{**A}\right)^{\alpha_A+\beta_A}}{\left(i^K\right)^{\beta_A}}\right)^{\frac{1}{1-\alpha_A-\beta_A}} \tag{154}$$

Manufactured exports demand:

$$x^M = y^{**}\left(p^{*MX}p^{**X}\right)^{-\theta^{**}} \tag{155}$$

Consumption inflation rate

$$\left(\pi^C\right)^{1-\theta_C} = \frac{a_D}{a_D + a_N\left(p^N\right)^{1-\theta_C}}\left(\pi\right)^{1-\theta_C} + \left(1 - \frac{a_D}{a_D + a_N\left(p^N\right)^{1-\theta_C}}\right)\left(\pi^N\right)^{1-\theta_C} \tag{156}$$

Bank real cost:

$$C^B = \frac{1}{2}\left[a_L^B\ell^2 + a_D^B d^2 - 2a_0^B \ell d\right] \tag{157}$$

Consumption relative price:

$$p^C = \left[a_D + (1-a_D)\left(p^N\right)^{1-\theta_C}\right]^{\frac{1}{1-\theta_C}}. \tag{158}$$

Investment relative price:

$$p^V = \left[b_D + (1-b_D)\left(p^N\right)^{1-\theta_V}\right]^{\frac{1}{1-\theta_V}}. \tag{159}$$

Policy equations:

Pure Exchange rate Crawl regimes (PEC)

$$b^{CB} = b_0^C.$$
$$\frac{\pi}{\pi^{**N}} = \delta^T.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$\begin{aligned}(1+i)^{1-h_0} &= \left(\Xi^{TR}\right)^{1-h_0}\left(\frac{\pi}{\pi^T}\right)^{h_1}\left(\frac{\pi^C}{\pi^{CT}}\right)^{h_2}\left(\frac{y}{y^T}\right)^{h_3}\left(\frac{e}{e^T}\right)^{h_4} \quad (160)\\ \Xi^{TR} &\equiv \frac{\pi^T}{\pi}\left\{1+\frac{\pi}{\pi^{**N}}\left[\left(1+i^{**}\right)\varphi_B\left(b^{*B}e\right)-1\right]\right\}\end{aligned}$$

$$r^{*CB} = r_0.$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$(1+i)^{1-h_0} = \left(\Xi^{TR}\right)^{1-h_0}\left(\frac{\pi}{\pi^T}\right)^{h_1}\left(\frac{\pi^C}{\pi^{CT}}\right)^{h_2}\left(\frac{y}{y^T}\right)^{h_3}\left(\frac{e}{e^T}\right)^{h_4}$$
$$r^{*CB} = \gamma^T\frac{m^{0,H}+d+\left(1-\gamma^{FX}\right)eb^{*B}}{e}$$

A first glance at these equations shows that several of the steady state variables are readily determined. This is the case of $\widetilde{w}$, $\widetilde{p}$, $\widetilde{p}^N$, $\widetilde{p}^{*X}$, which are all equal to one, implying that in the steady state there is no distinction between agents that optimize and those that index. Also, from (142)) we have the real marginal cost in the domestic sector equal to the inverse of the markup factor:

$$mc = \frac{\theta-1}{\theta} \equiv s_\theta^{-1}.$$

Also, the two pairs of equations that follow (142) yield the MITT and the MXTT as functions of the real exchange rate:

$$p^N = s_{\theta_N}e, \qquad \left(s_{\theta_N} \equiv \frac{\theta_N}{\theta_N-1}\right) \quad (161)$$

$$p^{*MX} = \frac{s_{\theta^*}}{e}, \qquad \left(s_{\theta^*} \equiv \frac{\theta^*}{\theta^*-1}\right) \quad (162)$$

Hence, using (161) in (158) and (159) and (162) in (155), the SS consumption and investment relative prices (in terms of domestic goods) and the manufactured export demand, are also functions of the real exchange rate:

$$p^C = \left[a_D + (1-a_D)\left(s_{\theta_N}e\right)^{1-\theta_C}\right]^{\frac{1}{1-\theta_C}}$$

$$p^V = \left[b_D + (1-b_D)\left(s_{\theta_N}e\right)^{1-\theta_V}\right]^{\frac{1}{1-\theta_V}}$$

$$x^M = y^{**}\left(\frac{s_{\theta^*}}{e}p^{**X}\right)^{-\theta^{**}}.$$

Expressions (145)-(148) yield:

$$\pi = \pi^N = \delta\pi^{**N}, \qquad \pi^w = \pi\mu^{z**}, \qquad \pi^{*MX} = \pi^{**N}. \tag{163}$$

The first equality, along with (156), implies that $\pi^C = \pi = \pi^N$. In the case of a Pure Crawl regime, the steady state gross rate of crawl is a target variable: $\delta = \delta^T$. Hence, we obtain the basic endogenous rates of inflation expressed in terms of exogenous variables:

$$\pi = \pi^N = \pi^C = \delta^T\pi^{**N}, \quad \pi^w = \delta^T\pi^{**N}\mu^{z**} \text{ and } \pi^{*MX} = \pi^{**N}. \tag{164}$$

Therefore, (139) implies that the steady state deposit rate is:

$$1 + i^D = \frac{\delta^T\pi^{**N}\mu^{z**}}{\beta} = \frac{\pi^w}{\beta}.$$

And in the case of Inflation Targeting, first notice that an obvious restriction for any (transitional) target on output, inflation, or the RER is that they converge to y , π, and e, respectively. Hence, introducing $y^T = y$, $e^T = e$, and (156) and (163) in (160) as well as the obvious consistency requirement $\pi^{CT} = \pi^T$(if both target variables are used simultaneously), simplifies the feedback rule to:

$$(1+i)^{1-h_0} = \left(\Xi^{TR}\right)^{1-h_0}\left(\frac{\pi}{\pi^T}\right)^{h_1+h_2}. \tag{166}$$

Notice that we have defined the constant Ξ^{TR} is such a way that using (148) and the steady state of the uncovered interest parity condition (149) yields

$$\Xi^{TR} = (1+i)\frac{\pi^T}{\pi}$$

Introducing this in (166) and recalling that $h_1+h_2 > 1$ (and hence $h_0+h_1+h_2 > 1$) gives $\pi = \pi^T$. Therefore, using this in (163) we again have all the steady state inflation rates in terms of exogenous variables:

$$\pi = \pi^N = \pi^C = \delta\pi^{**N} = \pi^T, \quad \pi^w = \pi^T\mu^{z**} \quad \text{and} \quad \pi^{*MX} = \pi^{**N}.$$

We now let δ^T stand for π^T/π^{**N} in the case of IT regimes. Hence, from now on we can use the same notation in all regimes: δ^T for the steady state rate of nominal depreciation, and π^T for the steady state rate of inflation.

Using (139) we verify that, as in the case of PEC regimes, the steady state gross deposit rate under IT regimes is also equal to the wage inflation rate divided by the time discount factor:

$$1 + i^D = \frac{\pi^T\mu^{z**}}{\beta} = \frac{\pi^w}{\beta}.$$

Therefore, we can use the symbols $\widetilde{\tau}_M$, $\widetilde{\varphi}_M$, and ϖ, as shorthand for $\widetilde{\tau}_M\left(\pi^T\mu^{z**}/\beta\right)$, $\widetilde{\varphi}_M\left(\pi^T\mu^{z**}/\beta\right)$, and $\mathcal{L}\left(\pi^T\mu^{z**}/\beta\right)$, respectively. Hence, from (137) and (138) we obtain the steady state value of i^K as:

$$i^K = \left(\mu^{z**}/\beta - 1 + \delta^K\right)\widetilde{\varphi}_M \equiv s_0.$$

And introducing this in (154), the primary goods supply is also a function of the RER:

$$a = \left(\kappa_A (s_0)^{-\beta_A} \left(ep^{**A}\right)^{\alpha_A+\beta_A}\right)^{\frac{1}{1-\alpha_A-\beta_A}}.$$

The rest of the SS system can be solved for the SS values of the remaining variables. However, this is unnecessary since it is convenient to use an array of great ratios (to GDP) in the calibration process, as we detail in a forthcoming paper.

14. Stochastic shocks

14.1 Permanent productivity shocks

In the shock specification for domestic output we follow ALLV (2005) in having a permanent productivity shock z_t, and a transitory productivity shock ϵ_t. However, we have two different versions for their dynamics. The first is similar to ALLV (2005) in postulating an AR(1) process for the deviation between the SOE and the LRW in total factor productivity levels $z_t^{\circ} \equiv z_t^{**}/z_t$ (which ALLV (2005) call "asymmetric technology shock"):

$$\widehat{z}_t^{\circ} = \rho^{z**}\widehat{z}_{t-1}^{\circ} + \varepsilon_t^{z^{\circ}}. \tag{168}$$

As seen in the previous section, we follow ALLV (2005) in assuming that in the SS total factor productivity levels and growth rates in the LRW and the SOE are equal ($z^{\circ} = 1$, and $\mu^z = \mu^{z**}$). Hence, $\widehat{z}_{t-1}^{\circ} = \log z_{t-1}^{\circ}$. In the second version, we assume a cointegrating relation between the logs of the technology shocks in the LRW and the SOE which includes a direct lagged influence of the LRW's rate of technological growth on that of the SOE. This is closer in spirit to the procedure in Juillard, Kamenik, Kumhof and Laxton (2005). In this second version we assume the following processes hold:

$$\widehat{\mu}_t^{z**} = \rho^{z**}\widehat{\mu}_{t-1}^{z**} + \varepsilon_t^{z**}, \tag{169}$$

$$\widehat{\mu}_t^{z} = \rho^{z}\widehat{\mu}_{t-1}^{z} + a_z\widehat{\mu}_{t-1}^{z**} + \alpha_z\widehat{z}_{t-1}^{\circ} + \varepsilon_t^{z}, \tag{170}$$

where ε_t^{z**} and ε_t^{z} are i.i.d. technology shocks. Putting these expressions in matrix form, with the drift terms in a constant vector C, perhaps reflects the cointegration assumption more transparently. We have:

$$\Delta \log \widetilde{z}_t = \alpha_z A \log \widetilde{z}_{t-1} + B\left(\Delta \log \widetilde{z}_{t-1}\right) + C + \widetilde{\varepsilon}_t,$$

where:

$$\widetilde{z}_t \equiv \begin{bmatrix} z_t^{**} \\ z_t \end{bmatrix}, \qquad \widetilde{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_t^{z**} \\ \varepsilon_t^{z} \end{bmatrix}$$

$$\begin{aligned} A &\equiv \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, \qquad B \equiv \begin{bmatrix} \rho^{z**} & 0 \\ a_z & \rho^z \end{bmatrix}, \\ C &\equiv \begin{bmatrix} \left(1-\rho^{z**}\right)\log \mu^{z**} \\ \left(1-\rho^{z}-a_z\right)\log \mu^{z**} \end{bmatrix}. \end{aligned}$$

and ρ^{z**}, $a_z + \rho^z \in (0, 1)$. During the transition, the growth rate of the LRW influences the growth rate of the SOE through the coefficient a_z, while the growth rate of the SOE has no influence on the rate of growth of the LRW. Also, the persistence coefficients may be different, and the disturbance terms may be correlated. Furthermore, notice that the law of motion for z_t° can be obtained from the following identity:

$$\widehat{z}_t^{\circ} - \widehat{z}_{t-1}^{\circ} = \widehat{\mu}_t^{z**} - \widehat{\mu}_t^{z}. \tag{171}$$

In this cointegration version we include (170) and (171) as model equations and (169) as an exogenous stochastic process. And in the non-cointegration version of the model we use (171) to eliminate $\widehat{\mu}_t^z$ and include (168) for the dynamics of the exogenous "asymmetric productivity shock" $\widehat{z}_t^{\circ}$.

14.2. Forcing stochastic processes

We stack the exogenous variables in a vector $Z_t^R \equiv (Z_t' \;\; P_{R,t}')'$ which is composed of a subvector which is independent on the monetary policy regime:

$$\begin{aligned} Z_t \;\equiv\; & [\widehat{\varepsilon}_t^z \;\; \widehat{\epsilon}_t \;\; \widehat{z}_t^C \;\; \widehat{z}_t^V \;\; \widehat{z}_t^H \;\; \widehat{z}_t^M \;\; \widehat{\varsigma}_t^K \;\; \widehat{\varsigma}_t^W \;\; \widehat{\varsigma}_t^A \;\; \widehat{\varsigma}_t^N \;\; \widehat{\gamma}_t^B \;\; \widehat{\gamma}_t^R \;\; \widehat{\ell}_t^G \;\; \widehat{g}_t \;\; \widehat{t}_t \\ & \widehat{\mu}_t^{z**} \;\; \widehat{\pi}_t^{**N} \;\; \widehat{p}_t^{**X} \;\; \widehat{p}_t^{**A} \;\; \widehat{y}_t^{**} \;\; \widehat{i}_t^{**} \;\; \widehat{\phi}_t^{**B}]' \end{aligned}$$

(where in the version without cointegration $\widehat{\varepsilon}_t^z$ must be replaced by $\widehat{z}_t^0$) and a subvector $P_{R,t}$ which depends on the policy regime ($R = PEC, IT$). Under a PEC regime:

$$P_{PEC,t} \equiv [\widehat{\delta}_t^T],$$

and under an IT regime:

$$P_{IT,t} \equiv [\widehat{\pi}_t^T \;\; \widehat{\pi}_t^{CT} \;\; \widehat{y}_t^T \;\; \widehat{e}_t^T]'.$$

In our baseline model (which addresses IT-MEF and no cointegration) we simply assume constant targets, so we only need Z_t. We can express the dynamics for the non-policy exogenous forcing variables in the form of a first order VAR process:

$$Z_t = MZ_{t-1} + \varkappa_t, \qquad \varkappa_t \sim iid\; N(0, \Sigma), \tag{172}$$

where M is a square matrix that is congruent with Z_t and has all its eigenvalues inside the unit circle.

If we have no reason to believe that the exogenous shocks are correlated, we can assume that they follow individual AR(1) processes. In particular, we can assume that the stochastic process for the logs of ϵ_t, z_t^H, z_t^C, z_t^V, z_t^M, y_t^{**}, and ς_t^q are AR(1):

$$\begin{aligned} \epsilon_t &= (\epsilon_{t-1})^{\rho_\epsilon}\, \varepsilon_t^{\epsilon} \\ z_t^n &= \left(z_{t-1}^n\right)^{\rho_n} \varepsilon_t^n, \qquad && n = H, C, V, M \\ \varsigma_t^q &= \left(\varsigma_{t-1}^q\right)^{\rho_{\varsigma^q}} \varepsilon_t^{\varsigma^q}, \qquad && q = W, K, A, N \\ y_t^{**} &= \left(y_{t-1}^{**}\right)^{\rho_{y^{**}}} \varepsilon_t^{q^*}, \end{aligned}$$

where all the persistence parameters (ρ^i) are positive and less than one, and the ε_t^i, are i.i.d. shocks. We assume that the steady state values ϵ, z^H, z^C, z^V are all unity. In the baseline model we take the Central Bank target rates of inflation π_t^T, π_t^{CT} as constants and we omit the other transitional targets (y_t^T and $\widehat{e}_t^T$). Therefore, $\widehat{\pi}_t^T$, π_t^{CT}, $\widehat{y}_t^T$ and $\widehat{e}_t^T$ disappear from the interest rate feedback policy rules (180) when we implement the baseline model.

15. Functional forms for auxiliary functions

The specific functional forms we use for the abnormal capital utilization cost, investment adjustment cost, and transaction cost functions are the following:

$$\tau_u(u_t) \equiv \frac{i^K}{a_u+1}\left(u_t^{a_u+1}-1\right), \qquad a_u>0 \tag{173}$$

$$\tau_V\left(\mu_t^V\right) \equiv \frac{a_V}{2}\left(\mu_t^V-\mu^z\right)^2, \qquad a_V>0 \tag{174}$$

$$\tau_M\left(\varpi_t\right) \equiv a_M z_t^M \varpi_t + \varpi_t^{-b_M} - c_M, \qquad a_M, b_M, c_M > 0. \tag{175}$$

According to (173) and (23), the utilization intensity of physical capital as a function of the rental rate is:

$$u_t = \left(\frac{i_t^K}{i^K}\right)^{\frac{1}{a_u}} \tag{176}$$

and the real return from renting one unit of capital (gross of depreciation) is:

$$\Gamma^K\left(i_t^K\right) \equiv \frac{i_t^K}{a_u+1}\left\{a_u\left(\frac{i_t^K}{i^K}\right)^{1+\frac{1}{a_u}}+1\right\}.$$

Hence, in the SS $u=1$ and $\Gamma^K\left(i^K\right)=i^K$. It is readily verified that (173) satisfies the theoretical assumption (3).

In the case of the investment adjustment cost we previously defined the rate of growth of real investment expenditure $\mu_t^V \equiv V_t/V_{t-1} = \left(v_t/v_{t-1}\right)\mu_t^z$, which is μ^{z**} in the SS. Hence, (174) complies with the theoretical assumption (174).

In the case of transaction costs, we use a modification of the functional form used in Uribe and Schmitt-Grohé (2003). There is a satiation level of cash/absorption after which the function becomes increasing in its argument. Obviously, only the decreasing portion of the function is relevant. We have three parameters for calibration: a_M, b_M, c_M.[14] We have also included a (negative) transactions technology shock z_t^M. An increase in z_t^M raises transactions costs for any given cash/absorption ratio ϖ_t. According to (25), the resulting liquidity preference function is:

$$\varpi_t = \frac{m_t^{0,H}}{p_t^C c_t + p_t^V v_t} = \mathcal{L}\left(1+i_t^D\right) \equiv \left[\frac{b_M}{a_M z_t^M + 1 - \frac{1}{1+i_t^D}}\right]^{\frac{1}{1+b_M}}. \tag{177}$$

Hence, household money demand is decreasing with respect to the deposit rate and increasing (with unit elasticity) with respect to private absorption. Also, a transactions cost shock diminishes household demand for cash as a ratio of household absorption (given the deposit interest rate). The actual effect on transactions costs $\tau_M\left(\varpi_t\right)$ depends on the elasticity of this function with respect to z_t^M. It is readily verified that an increase in z_t^M raises $z_t^M\varpi_t$, making actual transactions costs increase even more than this partial effect because the second term in (175) $\varpi_t^{-b_M}$ also increases. Using (25), the elasticity of household cash demand (as a

[14] All three parameters in the transaction cost function are useful in the calibration process, as we detail in a forthcoming paper.

fraction of absorption) with respect to the gross deposit rate is independent of the transactions costs shock:

$$\varepsilon_{m^{0,H},t} = \frac{\varpi_t^{1+b_M}}{(1+b_M)\, b_M\, (1+i_t^D)}.$$

And the resulting auxiliary function for the total effect on expenditure of a marginal increase in absorption (22) is also independent of the transactions costs shock:

$$\varphi_M(\varpi_t) = 1 - c_M + (1+b_M)\,\varpi_t^{-b_M}.$$

For the bank risk premium we use the following functional form:

$$p_B\left(e_t b_t^{*B}\right) \equiv \alpha_1^{RP}\left(e_t b_t^{*B}\right)^{\alpha_2^{RP}}, \qquad \alpha_1^{RP} > 0, \alpha_2^{RP} > 1.$$

Hence, in the risk adjusted uncovered interest parity (97) we have:

$$\varphi_B\left(e_t b_t^{*B}\right) = 1 + \phi_t^{B*} + \left(\alpha_2^{RP}+1\right)\alpha_1^{RP}\left(e_t b_t^{*B}\right)^{\alpha_2^{RP}}.$$

If we want to track the Government foreign debt (which decomposes from the rest of the system), we can calibrate its risk premium using the same functional form as for banks.

16. The log-linear systems

In this section we list the log-linear approximation of the system equations as they appear in section 12 after eliminating δ_t and collapsing the fiscal and balance of payments equations as there indicated. We introduce $\widehat{\mu}_t^z$ and $\widehat{z}_t^\circ$ as new variables, along with the growth dynamics equation (170) and the identity (171). The detailed log-linearization of the Phillips equations for domestic goods and wages (the most cumbersome) are in Appendix 1. And the definitions of the equation coefficients are in Appendix 2. For convenience, we change the order of the equations, listing first the static equations first, then the non-policy dynamic equations with no expectational terms second, then the non-policy dynamic equations with expectational terms, and finally the policy equations (which are dynamic and in the baseline version contain no expectational terms).

16.1. The equations

The log-linear equations of our systems are the following:

Static equations

Deposit market clearing:

$$\alpha_{DS}^B \widehat{d}_t + \left(1-\alpha_{DS}^B\right)\left[\alpha_1^{MD}\widehat{i}_t^D - \alpha_2^{MD}\widehat{i}_t + \alpha_3^{MD}\left(\alpha_B^{MD}\widehat{\gamma}_t^B + \alpha_R^{MD}\widehat{\gamma}_t^R\right)\right]$$
$$= \left(1+\alpha^{ML}\right)\widehat{i}_t^L - \alpha^{ML}\widehat{i}_t$$

Interbank cum Central Bank bond market clearing:

$$\alpha_A^B \widehat{\ell}_t + \left(1-\alpha_A^B\right)\widehat{b}_t^{CB} = \alpha_L^B\left(\widehat{d}_t - \alpha_B^{MD}\widehat{\gamma}_t^B - \alpha_R^{MD}\widehat{\gamma}_t^R\right) + \left(1-\alpha_L^B\right)\left(\widehat{e}_t + \widehat{b}_t^{*B}\right)$$

Cash market clearing:

$$\begin{aligned}\widehat{m}_t^0 &= a^{CM}\left[\alpha_A^{CM}\left(\widehat{c}_t+\widehat{p}_t^C\right)+\left(1-\alpha_A^{CM}\right)\left(\widehat{v}_t+\widehat{p}_t^V\right)-\alpha_D^{CM}\widehat{i}_t^D-\alpha_M^{CM}\widehat{z}_t^M\right]\\&\quad+\left(1-a^{CM}\right)\left(\widehat{d}_t+\widehat{\gamma}_t^B\right)\end{aligned}$$

Real GDP:

$$\begin{aligned}a^Y\widehat{y}_t &= \alpha_c^Y\left(\widehat{c}_t+\widehat{p}_t^C\right)+\alpha_v^Y\left(\widehat{v}_t+\widehat{p}_t^V\right)+\left(1-\alpha_c^Y-\alpha_v^Y-\alpha_{xm}^Y-\alpha_{xa}^Y\right)\widehat{g}_t\\&\quad+\alpha_{xm}^Y\widehat{x}_t^M+\alpha_{xa}^Y\left(\widehat{e}_t+\widehat{p}_t^{**A}+\widehat{x}_t^A\right)-\left(1-a^Y\right)\left(\widehat{p}_t^N+\widehat{n}_t\right).\end{aligned}$$

Intermediate demand for domestic goods:

$$\begin{aligned}\widehat{q}_t^{ID} &= a_1^{qD}\left[\alpha_A^{CM}\left(\widehat{c}_t+\widehat{p}_t^C\right)+\left(1-\alpha_A^{CM}\right)\left(\widehat{v}_t+\widehat{p}_t^V\right)+b_\tau^Q\widehat{i}_t^D\right]+a_2^{qD}\widehat{C}_{t+1}^B\\&\quad+\left(1-a_1^{qD}-a_2^{qD}\right)\left(\widehat{a}_t+\widehat{e}_t+\widehat{p}_t^{**A}\right)+a_0^{qD}\widehat{i}_t^K\end{aligned}$$

Loan supply:

$$\begin{aligned}&\alpha_{LS}^B\widehat{\ell}_t+\left(1-\alpha_{LS}^B\right)\left[\alpha_1^{MD}\widehat{i}_t^D-\alpha_2^{MD}\widehat{i}_t+\alpha_3^{MD}\left(\alpha_B^{MD}\widehat{\gamma}_t^B+\alpha_R^{MD}\widehat{\gamma}_t^R\right)\right]\\=&\left(1+\alpha^{ML}\right)\widehat{i}_t^L-\alpha^{ML}\widehat{i}_t\end{aligned}$$

Central Bank balance sheet:

$$\widehat{r}_t^{*CB}=a_1^{CB}\widehat{m}_t^0+a_2^{CB}\left(\widehat{d}_t+\widehat{\gamma}_t^R\right)+\left(1-a_1^{CB}-a_2^{CB}\right)\widehat{b}_t^{CB}-\widehat{e}_t$$

Primary goods supply:

$$\widehat{a}_t=\gamma_1^a\left(\widehat{e}_t+\widehat{p}_t^{**A}\right)-\gamma_2^a\widehat{i}_t^K$$

Manufactured exports demand:

$$\widehat{x}_t^M=\widehat{z}_t^\circ+\widehat{q}_t^{**}-\theta^{**}\left(\widehat{p}_t^{*MX}+\widehat{p}_t^{**X}\right)$$

Consumer Price Inflation

$$\widehat{\pi}_t^C=a_{PC}\widehat{\pi}_t^N+\left(1-a_{PC}\right)\widehat{\pi}_t.$$

Bank real cost:

$$\widehat{C}_{t+1}^B=a^{BC}\widehat{\ell}_t+\left(2-a^{BC}\right)\widehat{d}_t.$$

Consumption relative price:

$$\widehat{p}_t^C=a_{PC}\widehat{p}_t^N$$

Investment relative price:

$$\widehat{p}_t^V=a_{PV}\widehat{p}_t^N$$

Dynamic non-policy equations with no expectational terms:

Identities:

$$\widehat{\widehat{w}}_t - \widehat{\widehat{w}}_{t-1} = \widehat{\pi}^w_t - \widehat{\pi}_t - \widehat{\mu}^z_t$$
$$\widehat{p}^N_t - \widehat{p}^N_{t-1} = \widehat{\pi}^N_t - \widehat{\pi}_t$$
$$\widehat{p}^{*XM}_t - \widehat{p}^{*XM}_{t-1} = \widehat{\pi}^{*XM}_t - \widehat{\pi}^{**N}_t$$
$$\widehat{z}^{\circ}_t - \widehat{z}^{\circ}_{t-1} = \widehat{\mu}^{z**}_t - \widehat{\mu}^z_t.$$

Growth:

$$\widehat{\mu}^z_t = \rho^z \widehat{\mu}^z_{t-1} + a_z \widehat{\mu}^{z**}_{t-1} + \alpha_z \widehat{z}^{\circ}_{t-1} + \varepsilon^z_t,$$

Central Bank quasi-fiscal surplus:

$$\widehat{qf}_t = a^{QF}\left\{\alpha^{QF}_r \widehat{i}^{**}_{t-1} + \left(1-\alpha^{QF}_r\right)\left[\widehat{\pi}^{**N}_t - \widehat{\pi}_t - (\widehat{e}_t - \widehat{e}_{t-1})\right]\right.$$
$$\left. + \widehat{r}^{*CB}_{t-1} + \widehat{e}_t - \widehat{\pi}^{**N}_t - \widehat{\mu}^z_t\right\} + \left(1-a^{QF}\right)\left[\left(i^{-1}+1\right)\widehat{i}_t + b^{CB}_{t-1} - \widehat{\pi}_t - \widehat{\mu}^z_t\right]$$

Government domestic surplus:

$$\widehat{gd}_t = a^{GD}_1 \widehat{t}_t - a^{GD}_2 \widehat{g}_t + a^{GD}_3 \widehat{\ell}^G_t - \left(a^{GD}_1 - a^{GD}_2 + a^{GD}_3 - 1\right)\left(\widehat{i}^L_{t-1} + \widehat{\ell}^G_{t-1} - \widehat{\pi}_t - \widehat{\mu}^z_t\right)$$

Balance of Payments cum Fiscal:

$$a^{BPF}_1 \widehat{r}^{*CB}_t + a^{BPF}_2 [\widehat{i}^{**}_{t-1} + a^{RP}\widehat{\phi}^{**B}_{t-1} + \left(1-a^{RP}\right)\alpha^{RP}_2\left(\widehat{b}^{*B}_{t-1} + \widehat{e}_{t-1}\right) + \widehat{b}^{*B}_{t-1} - \widehat{\pi}^{**N}_t - \widehat{\mu}^z_t]$$
$$+ a^{BPF}_3 \widehat{qf}_t + \left(1 - a^{BPF}_1 - a^{BPF}_2 - a^{BPF}_3\right)\widehat{gd}_t - \left(1 - a^{BPF}_1 - a^{BPF}_3\right)\widehat{e}_t =$$
$$= a^{BPF}_4 \widehat{b}^{*B}_t + a^{BPF}_5\left(\widehat{r}^{*CB}_{t-1} - \widehat{\pi}_t - \widehat{\mu}^z_t\right) + \left(1 - a^{BPF}_4 - a^{BPF}_5\right)\widehat{tb}_t$$

Trade balance:

$$\widehat{tb}_t \equiv a^{TB}_1\left(\widehat{p}^{*MX}_t + \widehat{x}^M_t\right) + a^{TB}_2\left(\widehat{p}^{**A}_t + \widehat{x}^A_t\right) - \left(a^{TB}_1 + a^{TB}_2 - 1\right)\widehat{n}_t$$

Physical capital accumulation:

$$\widehat{k}_{t+1} = a_K\left(\widehat{k}_t - \widehat{\mu}^z_t\right) + (1-a_K)\left(\widehat{v}_t + \widehat{z}^V_t\right)$$

Physical capital rental market clearing:

$$(1+1/a_u)\,\widehat{i}^K_t = \widehat{\mu}^z_t - \widehat{k}_t + \gamma^K\left[\widehat{mc}_t + \alpha_q\widehat{q}_t - \left(1+1/i^L\right)\alpha^{MC}_K \widehat{i}^L_{t-1} - \alpha^{MC}_K \widehat{\varsigma}^K_t\right]$$
$$+ \left(1-\gamma^K\right)\left(\widehat{e}_t + \widehat{p}^{**A}_t + \widehat{a}_t\right)$$

Labor market clearing:

$$\widehat{h}_t = \widehat{mc}_t + \alpha_q\widehat{q}_t - \widehat{\widehat{w}}_t - \left(1+1/i^L\right)\alpha^{MC}_W \widehat{i}^L_{t-1} - \alpha^{MC}_W \widehat{\varsigma}^W_t$$

Domestic goods market clearing:

$$\gamma^Q\widehat{q}_t = a^Q_y\widehat{y}_t + a^Q_D\widehat{q}^{ID}_t + \left(1 - a^Q_y - a^Q_D\right)\left[\widehat{mc}_t + \alpha_q\widehat{q}_t - \delta^{LM}_N\widehat{i}^L_{t-1} - \alpha^{MC}_N\widehat{\varsigma}^N_t\right]$$
$$- \left(1-\gamma^Q\right)\left(\widehat{e}_t + \widehat{p}^{**A}_t + \widehat{x}^A_t\right)$$

Real marginal cost:

$$\widehat{mc}_t = a^q \widehat{i}_t^K + b^q \widehat{\overline{w}}_t + c^q \left(\widehat{e}_t + \widehat{p}_t^{**A}\right) + \left(1 - a^q - b^q - c^q\right) \widehat{p}_t^N + \alpha_L^{MC} \widehat{i}_{t-1}^L$$
$$+ a^q \alpha_K^{MC} \widehat{\varsigma}_t^K + b^q \alpha_W^{MC} \widehat{\varsigma}_t^W + c^q \alpha_A^{MC} \widehat{\varsigma}_t^A + \left(1 - a^q - b^q - c^q\right) \alpha_N^{MC} \widehat{\varsigma}_t^N - \widehat{\epsilon}_t.$$

Import demand:

$$\widehat{n}_t = a_1^N \left(\widehat{c}_t + \widehat{p}_t^C\right) + a_2^N \left(\widehat{v}_t + \widehat{p}_t^V\right) - \widehat{p}_t^N$$
$$+ \left(1 - a_1^N - a_2^N\right) \left[\widehat{mc}_t + \alpha_q \widehat{q}_t - \left(1 + 1/i^L\right) \alpha_N^{MC} \widehat{i}_{t-1}^L - \alpha_N^{MC} \widehat{\varsigma}_t^N\right]$$

Primary exports:

$$\widehat{a}_t = \gamma^{XA} \left[\widehat{mc}_t + \alpha_q \widehat{q}_t - \left(1 + 1/i^L\right) \alpha_A^{MC} \widehat{i}_{t-1}^L - \alpha_A^{MC} \widehat{\varsigma}_t^A - \widehat{e}_t - \widehat{p}_t^{**A}\right] + \left(1 - \gamma^{XA}\right) \widehat{x}_t^A$$

Dynamic non-policy equations with expectational terms:

Consumption:

$$\left(1 + a_C\right) \left\{\widehat{\mu}_t^z + \widehat{z}_t^C - \left[\left(1 + \alpha_C\right) \left(\widehat{c}_t + \widehat{\mu}_t^z\right) - \alpha_C \widehat{c}_{t-1}\right]\right\}$$
$$- a_C \left\{E_t \widehat{z}_{t+1}^C - \left[\left(1 + \alpha_C\right) E_t \left(\widehat{c}_{t+1} + \widehat{\mu}_{t+1}^z\right) - \alpha_C \widehat{c}_t\right]\right\} = \widehat{\lambda}_t^{\circ} + \varepsilon_M \widehat{i}_t^D$$

Investment:

$$\widehat{\zeta}_t^{\circ} + \widehat{z}_t^V - a_V \left(\mu^z\right)^2 \left(\widehat{v}_t - \widehat{v}_{t-1} + \widehat{\mu}_t^z\right) + \beta a_V \left(\mu^z\right)^2 E_t \left(\widehat{v}_{t+1} - \widehat{v}_t + \widehat{\mu}_{t+1}^z\right)$$
$$= \widehat{\lambda}_t^{\circ} + \varepsilon_M \widehat{i}_t^D$$

Wage inflation Phillips equation:

$$\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w = \beta E_t \left(\widehat{\pi}_{t+1}^w - \widehat{\pi}_t^w\right) + \frac{\left(1 - \alpha_W\right)\left(1 - \beta \alpha_W\right)}{\alpha_W \left(1 + \psi \chi\right)} \left(\chi \widehat{h}_t + \widehat{z}_t^H - \widehat{\lambda}_t^{\circ} - \widehat{\overline{w}}_t\right).$$

Domestic inflation Phillips equation:

$$\widehat{\pi}_t - \widehat{\pi}_{t-1} = \beta \left(E_t \widehat{\pi}_{t+1} - \widehat{\pi}_t\right) + \frac{\left(1 - \alpha\right)\left(1 - \alpha \beta\right)}{\alpha} \widehat{mc}_t.$$

Imported goods inflation Phillips equation:

$$\widehat{\pi}_t^N - \widehat{\pi}_{t-1}^N = \beta \left(E_t \widehat{\pi}_{t+1}^N - \widehat{\pi}_t^N\right) + \frac{\left(1 - \alpha_N\right)\left(1 - \alpha_N \beta\right)}{\alpha_N} \left(\widehat{e}_t - \widehat{p}_t^N\right),$$

Manufactured exports inflation Phillips equation:

$$\widehat{\pi}_t^{*MX} - \widehat{\pi}_{t-1}^{*MX} = \beta \left(E_t \widehat{\pi}_{t+1}^{*MX} - \widehat{\pi}_t^{*MX}\right) - \frac{\left(1 - \alpha_X\right)\left(1 - \alpha_X \beta\right)}{\alpha_X} \left(\widehat{e}_t + \widehat{p}_t^{*MX}\right)$$

Marginal utility of installed physical capital:

$$\zeta_t^{\circ} = E_t \left\{\beta a_K \widehat{\zeta}_{t+1}^{\circ} + \left(1 - \beta a_K\right) \left[\left(\frac{2a_u + 1}{a_u + 1}\right) \widehat{i}_{t+1}^K + \widehat{\lambda}_{t+1}^{\circ}\right] - \widehat{\mu}_{t+1}^z\right\}$$

Marginal utility of real income:

$$\widehat{\lambda}_t^{\circ} = E_t\widehat{\lambda}_{t+1}^{\circ} + \widehat{i}_t^D - E_t\widehat{\pi}_{t+1} - E_t\widehat{\mu}_{t+1}^z$$

Bank arbitrage:

$$\begin{aligned}\widehat{i}_t &= \gamma_1^B\gamma_2^B\left[\widehat{i}_t^{**} + \overline{a}^{RP}\widehat{\phi}_t^{**B} + \left(1-\overline{a}^{RP}\right)\alpha_1^{RP}\left(\widehat{e}_t + \widehat{b}_t^{*B}\right)\right] \\ &\quad +\gamma_1^B\beta^B\left(E_t\widehat{e}_{t+1} - \widehat{e}_t + E_t\widehat{\pi}_{t+1} - E_t\widehat{\pi}_{t+1}^{**N}\right) \\ &\quad +\gamma_1^B\left(1-\beta^B\right)\left(\widehat{e}_t - \widehat{e}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^{**N}\right)\end{aligned}$$

Loan market clearing:

$$\begin{aligned}\widehat{\ell}_t = a^{LM}[E_t\widehat{mc}_{t+1} + E_t\widehat{\mu}_{t+1}^z + \alpha_q E_t\widehat{q}_{t+1} - \gamma^{LM}\widehat{i}_t^L + \delta_K^{LM}E_t\widehat{\varsigma}_{t+1}^K + \delta_W^{LM}E_t\widehat{\varsigma}_{t+1}^W \\ +\delta_A^{LM}E_t\widehat{\varsigma}_{t+1}^A + \delta_N^{LM}E_t\widehat{\varsigma}_{t+1}^N] + \left(1-a^{LM}\right)\widehat{\ell}_t^G\end{aligned} \quad (179)$$

Policy equations:

Pure Exchange rate Crawl regimes (PEC)

$$\widehat{b}_t^{CB} = 0$$

$$\widehat{e}_t - \widehat{e}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^{**N} = \widehat{\delta}_t^T.$$

Inflation Targeting under a Pure Exchange rate Float regime (IT-PEF)

$$\widehat{r}_t^{*CB} = 0,$$

$$\widehat{i}_t = h_0\widehat{i}_{t-1} + h_1\left(\widehat{\pi}_t - \widehat{\pi}_t^T\right) + h_2\left(\widehat{\pi}_t^C - \widehat{\pi}_t^{CT}\right) + h_3\left(\widehat{y}_t - \widehat{y}_t^T\right) + h_4\left(\widehat{e}_t - \widehat{e}_t^T\right) \quad (180)$$

Inflation Targeting under a Managed Exchange rate Float regime (IT-MEF)

$$\widehat{r}_t^{*CB} = k_0\widehat{r}_{t-1}^{*CB} - k_1\left(\widehat{e}_t - \widehat{e}_{t-1}\right). \quad (181)$$

$$\widehat{i}_t = h_0\widehat{i}_{t-1} + h_1\left(\widehat{\pi}_t - \widehat{\pi}_t^T\right) + h_2\left(\widehat{\pi}_t^C - \widehat{\pi}_t^{CT}\right) + h_3\left(\widehat{y}_t - \widehat{y}_t^T\right) + h_4\left(\widehat{e}_t - \widehat{e}_t^T\right).$$

In a forthcoming paper we calibrate and solve numerically the version of the model without cointegrated productivity shocks (due to its simplicity) and with the IT-MEF policy regime (due to its relevance). To obtain the system without cointegration we make only minor changes in the equations above. First, we eliminate the growth equation, which makes $\widehat{\varepsilon}_t^z$ disappears from the list of exogenous autorregressive processes, as well as the identity that involves $\widehat{z}_t^{\circ}$ because we use it to eliminate $\widehat{\mu}_t^z$ from all the remaining equations. Hence, $\widehat{z}_t^{\circ}$ replaces $\widehat{\varepsilon}_t^z$ in the list of exogenous autorregressive processes. Finally, we eliminate $\widehat{p}_t^C$, $\widehat{p}_t^V$, and $\widehat{C}_t^B$ in order to reduce the number of equations without significantly complicating the other equations. This leaves us with 33 equations and endogenous variables.

16.2 The log-linearized systems in matrix format

There are various ways to solve the system numerically. We have chosen to organize the equations in a way suitable for the application of the generalized Schur (or QZ) decomposition (see Klein (2000)). Another possibility would be to use the Anderson-Moore algorithm (see Anderson (2000)), which does not require manipulating the variables or equations in any way. Alternative and related methods are described in Blanchard and Kahn (1980), Binder and Pesaran (1995), King and Watson (1998), Uhlig (1999), and Sims (2000).

To put the dynamic system in matrix form we have found convenient to stack the endogenous variables in three vectors (W_t, X_t, Y_t):

$$W_t \equiv [\widehat{i}_t^D \;\; \widehat{b}_t^{CB} \;\; \widehat{m}_t^0 \;\; \widehat{y}_t \;\; \widehat{q}_t^D \;\; \widehat{\ell}_t \;\; \widehat{d}_t \;\; \widehat{a}_t \;\; \widehat{x}_t^M \;\; \widehat{\pi}_t^C]'$$

$$X_t \equiv [\widehat{\overline{w}}_t \;\; \widehat{p}_t^N \;\; \widehat{p}_t^{*X} \;\; \widehat{b}_t^{*B} \;\; \widehat{k}_{t+1} \;\; \widehat{i}_t^K \;\; \widehat{h}_t \;\; \widehat{q}_t \;\; \widehat{mc}_t \;\; \widehat{n}_t \;\; \widehat{x}_t^A \;\; \widehat{r}_t^{*CB} \;\; \widehat{i}_t]'$$

$$Y_t \equiv [\widehat{c}_t \;\; \widehat{v}_t \;\; \widehat{\pi}_t^w \;\; \widehat{\pi}_t \;\; \widehat{\pi}_t^N \;\; \widehat{\pi}_t^{*MX} \;\; \widehat{e}_t \;\; \widehat{\overset{\circ}{\zeta}}_t \;\; \widehat{\overset{\circ}{\lambda}}_t \;\; \widehat{i}_t^L]',$$

W_t includes the variables that could most naturally (or conveniently) be eliminated by means of the static equations. However, we maintain all the variables throughout, particularly because this vector includes two possible target variables in the feedback rules ($\widehat{y}_t$ and $\widehat{\pi}_t^C$), but also to maintain the complete set of variables for obtaining the impulse-response functions. X_t includes the variables whose associated equations do not contain expectational terms. Notice that our two policy instruments are the last two elements in this vector (given that the baseline version of the model does not include forward looking feedback rules). And Y_t includes the variables whose associated equations contain expectational terms. We can express the three blocks of equations (static, dynamic with no expectational terms, and dynamic with expectational terms) in the following matrix structural form:

$$H_w W_t = H^x X_t + H^y Y_t + H^z Z_t \tag{182}$$

$$\begin{aligned} B_{11}X_t + B_{12}Y_t &= C_{11}X_{t-1} + C_{12}Y_{t-1} + D_1 W_t \\ &\quad + J_{11}Z_t + J_{12}Z_{t-1} \end{aligned} \tag{183}$$

$$\begin{aligned} B_{21}X_t + B_{22}Y_t &= A_{21}E_tX_{t+1} + A_{22}E_tY_{t+1} + C_{22}Y_{t-1} + D_2W_t \\ &\quad + J_{21}Z_t + J_{22}Z_{t-1} + J_{20}E_tZ_{t+1}. \end{aligned} \tag{184}$$

Using the fact that $E_tZ_{t+1} = MZ_t$ These equations can be conveniently stacked as:

$$\begin{bmatrix} H_w & -H^x & -H^y \\ -D_1 & B_{11} & B_{12} \\ -D_2 & B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} W_t \\ X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} E_tW_{t+1} \\ E_tX_{t+1} \\ E_tY_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ C_{11} & C_{12} \\ 0 & C_{22} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ Y_{t-1} \end{bmatrix} + \begin{bmatrix} H^z & 0 \\ J_{11} & J_{12} \\ J_{21} + MJ_{20} & J_{22} \end{bmatrix} \begin{bmatrix} Z_t \\ Z_{t-1} \end{bmatrix} \tag{185}$$

Next we define new variables in order to put the system in state space form. None of the variables in W appear lagged in the system. Since only a subset of the variables in vectors X_t and Y_t actually appear lagged (in at least one equation), we define selector matrices S_X and S_Y that select only the elements of X_t and Y_t that appear lagged, and define new, lower dimensional, vectors $\overline{X}_t = S_X X_{t-1}$ and $\overline{Y}_t = S_Y Y_{t-1}$. We also define matrices $\overline{C}_{j1} = C_{j1}S'_X$, $\overline{C}_{j2} = C_{j2}S'_Y$ $(j = 1, 2)$ that have the same elements as matrices C_{ij} but leave out the columns of zeros. Notice that, by construction, $\overline{C}_{11}\overline{X}_t = C_{11}X_t$ and $\overline{C}_{j2}\overline{Y}_t = C_{j2}Y_t$, $(j = 1, 2)$. We can hence express the preceding equations in state space form as:

$$\begin{bmatrix} I_{7\times 7} & 0 & 0 & 0 & 0 \\ 0 & I_{8\times 8} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \overline{X}_{t+1} \\ \overline{Y}_{t+1} \\ E_t W_{t+1} \\ E_t X_{t+1} \\ E_t Y_{t+1} \end{bmatrix} = \tag{186}$$

$$= \begin{bmatrix} 0 & 0 & 0 & S_X & 0 \\ 0 & 0 & 0 & 0 & S_Y \\ 0 & 0 & H_w & -H^x & -H^y \\ -\overline{C}_{11} & -\overline{C}_{12} & -D_1 & B_{11} & B_{12} \\ 0 & -\overline{C}_{22} & -D_2 & B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \overline{X}_t \\ \overline{Y}_t \\ W_t \\ X_t \\ Y_t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -H^z & 0 \\ -J_{11} & -J_{12} \\ -\left(J_{21} + MJ_{20}\right) & -J_{22} \end{bmatrix} \begin{bmatrix} Z_t \\ Z_{t-1} \end{bmatrix}$$

where

$$\overline{X}_t \equiv [\widehat{\overline{w}}_{t-1} \;\; \widehat{p}^{N}_{t-1} \;\; \widehat{p}^{*X}_{t-1} \;\; \widehat{b}^{*B}_{t-1} \;\; \widehat{k}_t \;\; \widehat{r}^{*CB}_{t-1} \;\; \widehat{\imath}_{t-1}]'$$

$$\overline{Y}_t \equiv [\widehat{c}_{t-1} \;\; \widehat{v}_{t-1} \;\; \widehat{\pi}^{w}_{t-1} \;\; \widehat{\pi}_{t-1} \;\; \widehat{\pi}^{N}_{t-1} \;\; \widehat{\pi}^{*MX}_{t-1} \;\; \widehat{e}_{t-1} \;\; \widehat{\imath}^{L}_{t-1}]'$$

$$\begin{aligned} S_X &\equiv \begin{bmatrix} I_{5\times 5} & 0_{5\times 6} & 0_{5\times 2} \\ 0_{2\times 5} & 0_{2\times 6} & I_{2\times 2} \end{bmatrix} \\ S_X &\equiv \begin{bmatrix} I_{7\times 7} & 0_{7\times 2} & 0_{7\times 1} \\ 0_{1\times 7} & 0_{1\times 2} & 1 \end{bmatrix}. \end{aligned}$$

Also, the first order autorregressive equation for the forcing processes can be expressed as:

$$\begin{bmatrix} Z_t \\ Z_{t-1} \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-2} \end{bmatrix} + \begin{bmatrix} \varkappa_t \\ \varkappa_{t-1} \end{bmatrix}. \tag{187}$$

In compact (and obvious) notation, (186) and (187) have the format necessary to apply the solution method in Klein (2000):

$$\begin{aligned} \widetilde{A}E_t\widetilde{X}_{t+1} &= \widetilde{B}\widetilde{X}_t + \widetilde{C}\widetilde{Z}_t \\ \widetilde{Z}_t &= \widetilde{M}\widetilde{Z}_{t-1} + \widetilde{\varkappa}_t. \end{aligned}$$

Notice that since $\widetilde{A}$ is clearly singular, the traditional Blanchard and Kahn (1980) method cannot be used (at least in this particular state space setup). The generalized Schur decomposition (also called QZ decomposition) is appropriate for this case. As long as there exists some complex ω such that $det(\widetilde{A}\omega - \widetilde{B}) \neq 0$, there exist unitary matrices[15] of complex numbers Q and Z such that $Q\widetilde{A}Z \equiv S$ and $Q\widetilde{B}Z \equiv T$ are upper triangular and such that for all i the diagonal elements S_{ii} and T_{ii} are not both zero. Also, the set of generalized eigenvalues is the set of ratios T_{ii}/S_{ii} (where, with abuse of language, when $S_{ii} = 0$ we call the corresponding generalized eigenvalue "infinite"). Furthermore, the pairs (S_{ii}, T_{ii}) can be arranged in any order. Hence the eigenvalues can be arranged so that the ones within the unit disk come first. We also partition $\widetilde{X}_t$ into two parts such that all the predetermined variables come first. The state variables (which are the nk=15 that appear lagged in the system) are included in a vector $\widetilde{k}_t$, and the remaining endogenous variables in a vector $\widetilde{d}_t$:

$$\widetilde{X}_t = \begin{bmatrix} \overline{X}_t \\ \overline{Y}_t \\ W_t \\ X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \widetilde{k}_t \\ \widetilde{d}_t \end{bmatrix}, \qquad \text{where } \widetilde{k}_t = \begin{bmatrix} \overline{X}_t \\ \overline{Y}_t \end{bmatrix}, \ \widetilde{d}_t = \begin{bmatrix} W_t \\ X_t \\ Y_t \end{bmatrix}.$$

Klein (2000) proves that if the resulting (after rearrangement) upper left block Z_{11} of Z (which includes its first nk rows and columns) is non-singular, and the number of generalized eigenvalues within the unit circle (i.e., the number of $i \in \{1, ..., nx\}$ such that $|T_{ii}| < |S_{ii}|$, where nx=15+33=48 is the dimension of $\widetilde{X}_t$) is equal to the dimension nk=15 of $\widetilde{k}_t$, then for any given k_0 there exists a saddlepath solution (which is almost surely (P) unique) and can be expressed as:

$$\begin{aligned} \widetilde{k}_{t+1} &= G\widetilde{k}_t + H\widetilde{Z}_t + \xi_{t+1} \\ \widetilde{d}_t &= K\widetilde{k}_t + L\widetilde{Z}_t, \end{aligned}$$

where ξ_{t+1} is a martingale difference process,

$$\begin{aligned} G &= Z_{11}S_{11}^{-1}T_{11}Z_{11}^{-1} \\ H &= (GZ_{12} - Z_{11}S_{11}^{-1}T_{12})R + \left(Z_{11}S_{11}^{-1}S_{12} - Z_{12}\right) R\widetilde{M} + Z_{11}S_{11}^{-1}Q_1\widetilde{C} \\ K &= Z_{21}Z_{11}^{-1} \\ L &= (KZ_{12} - Z_{22})\, R \\ vec(R) &= \left[I - \widetilde{M} \otimes \left(T_{22}^{-1}S_{22}\right)\right]^{-1} vec\left(T_{22}^{-1}Q_2\widetilde{C}\right). \end{aligned}$$

and Q_1, Q_2 are the corresponding upper and lower blocks of Q.

To put the model solution in a state space form convenient for impulse response analysis we stack the exogenous autorregressive processes along with the model solution:

$$\begin{bmatrix} \widetilde{k}_{t+1} \\ \widetilde{Z}_{t+1} \end{bmatrix} = \begin{bmatrix} G & H \\ 0 & \widetilde{M} \end{bmatrix} \begin{bmatrix} \widetilde{k}_t \\ \widetilde{Z}_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \widetilde{\varkappa}_{t+1}$$

[15] A square matrix is unitary if its inverse is equal to its conjugate transpose. Hence, a unitary matrix is always invertible.

$$\left[\widetilde{d}_t\right] = \left[\begin{array}{cc} K & L \end{array}\right] \left[\begin{array}{c} \widetilde{k}_t \\ \widetilde{Z}_t \end{array}\right].$$

We now have the system in the form required for using MATLAB's (Control System Toolbox) LTI Viewer:

$$\begin{aligned} x(t+1) &= AA * x(t) + BB * z(t) \\ y(t) &= CC * x(t) + DD * z(t), \end{aligned}$$

where in our case

$$\begin{aligned} AA &\equiv \left[\begin{array}{cc} G & H \\ 0 & \widetilde{M} \end{array}\right], & BB &\equiv \left[\begin{array}{c} 0 \\ I \end{array}\right] \\ CC &= \left[\begin{array}{cc} K & L \end{array}\right], & DD &\equiv [0] \end{aligned}$$

Since we have 44 (=2*22) exogenous variables (or inputs, z) and 33 (=10+13+10) endogenous variables (or outputs y), we obtain 1452 different impulse response function (IRF) graphs, of which only half (i.e 726) are of interest (the rest pertain to shocks to the lagged exogenous variables in $\widetilde{Z}_t$). They are shown in Appendix 4.

17. Conclusion

This paper develops a relatively large rational expectations, dynamic and stochastic general equilibrium model for a small economy whose growth stems from a unit root technology shock that is cointegrated with the analogous technology shock in the rest of the world, although a simpler version without cointegration is also developed. The model has households, four types of firms (domestic, importing, exporting, and primary), banks and a public sector. The primary sector firms are perfectly competitive while the firms in the other three sectors are monopolistically competitive. There are also perfectly competitive banks (without entry or exit). Importing and exporting firms engage in local currency pricing. Households and firms in the domestic, manufacturing export and import sectors are monopolistic competitors that engage in sticky nominal wage or price setting. Consequently, the model has four Phillips inflation equations (for wage inflation, domestic goods inflation, imported goods inflation, and manufactured exports inflation, respectively). We use the Calvo-Rotemberg sticky pricing model complemented by the Yun-Christiano-Eichenbaum-Evans extension for full indexation to previous period inflation for price or wage setters that don't have the opportunity of optimizing. Households make the consumption and investment decisions and also decide on the intensity of utilization of the physical capital they rent to domestic and primary sector firms in a competitive market. They generate demand for cash, which is introduced through a stylized transactions costs function, and bank deposits. Banks finance a stochastic fraction of the domestic firms' wage bill, capital rental bill, primary inputs bill and imported inputs bill, as well as the Government's exogenous demand for loans. They also issue deposits and obtain funds abroad to finance their loans, hold cash and regulatory reserves, and purchase Central Bank (domestic currency denominated) bonds. Their profit maximization yields the model's risk adjusted uncovered interest parity equation. The Central Bank issues currency and domestic currency bonds, and holds foreign currency reserves and regulatory bank reserves.

The main focus in monetary policy matters is on building a framework that can contain alternative monetary policy regimes, including those where the Central Bank uses two simultaneous instruments. For this, the Central Bank's balance sheet and the interbank cum Central Bank bond market equilibrium play important roles. We include the extreme cases of a crawling exchange rate peg with a pure interest rate float and an inflation targeting regime with a pure exchange rate float, but focus primarily in an inflation targeting with managed float regime where the Central Bank simultaneously intervenes in the interbank cum Central Bank bond market and the foreign exchange market with two corresponding simple policy feedback rules.

We calibrated the model for Argentina and solved it numerically using Klein's (2000) method. We show the resulting impulse response functions in an Appendix. In future research we hope to transform the model to obtain the optimal feedback rules under commitment. In such an extension it will be interesting to see whether and to what extent it is optimal for the Central Bank to incur in foreign exchange market intervention.

Appendix 1: Log-linearization of the Phillips equations

Phillips equation for domestic goods

We first log-linearize the Phillips equations for domestic goods, since the procedure is simpler than with the one for the wage rate. We rewrite the equations to be log-linearized for the reader's convenience:

$$\pi_t^{1-\theta} = \alpha\pi_{t-1}^{1-\theta} + (1-\alpha)\left(\widetilde{p}_t\pi_t\right)^{1-\theta}. \tag{188}$$

$$0 = E_t\sum_{j=0}^{\infty}(\beta\alpha)^j\,\overline{\Lambda}^{\circ}_{t+j}q_{t+j}\left(\pi_{t+j}\right)^{\theta}\left\{\frac{\widetilde{p}_t\pi_t}{\pi_{t+j}} - \frac{\theta}{\theta-1}mc_{t+j}\right\}, \tag{189}$$

First we express these equations in terms of the log deviations from steady state values. For a variable π_t, for example, we define the log deviation from steady state as:

$$\widehat{\pi}_t \equiv \log\left(\frac{\pi_t}{\pi}\right).$$

Take (188) first. Dividing through by $\pi_t^{1-\theta}$ and taking logs yields:

$$0 = \log\left\{\alpha\left(\frac{\pi_{t-1}}{\pi_t}\right)^{1-\theta} + (1-\alpha)\widetilde{p}_t^{1-\theta}\right\}.$$

The steady state values for π_t and $\widetilde{p}_t$ are π and 1, respectively, so we can write this expression in terms of the ratios of the variables and their steady state values:

$$\begin{aligned}
0 &= \log\left\{\alpha\left(\frac{\pi_{t-1}/\pi}{\pi_t/\pi}\right)^{1-\theta} + (1-\alpha)\widetilde{p}_t^{1-\theta}\right\} \\
&= \log\left\{\alpha\exp\left[(1-\theta)\left(\log\frac{\pi_{t-1}}{\pi} - \log\frac{\pi_t}{\pi}\right)\right] + (1-\alpha)\exp\left[(1-\theta)\log\widetilde{p}_t\right]\right\} \\
&= \log\left\{\alpha\exp\left[(1-\theta)\left(\widehat{\pi}_{t-1} - \widehat{\pi}_t\right)\right] + (1-\alpha)\exp\left[(1-\theta)\,\widehat{\widetilde{p}}_t\right]\right\} \\
&\equiv G(\widehat{\pi}_t, \widehat{\pi}_{t-1}, \widehat{\widetilde{p}}_t).
\end{aligned}$$

Second, a linear approximation of this expression is:

$$G(\widehat{\pi}_t, \widehat{\pi}_{t-1}, \widehat{\widetilde{p}}_t) \simeq G + G_1 \widehat{\pi}_t + G_2 \widehat{\pi}_{t-1} + G_3 \widehat{\widetilde{p}}_t,$$

where G is the value of the function at the steady state values of the variables, and G_j is the partial derivative of G with respect to its jth variable, valued at the steady state values of the variables. Calculating the corresponding partial derivatives gives:

$$0 = -\alpha (1-\theta) (\widehat{\pi}_t - \widehat{\pi}_{t-1}) + (1-\alpha)(1-\theta) \widehat{\widetilde{p}}_t$$

and hence:

$$\widehat{\widetilde{p}}_t = \frac{\alpha}{1-\alpha} (\widehat{\pi}_t - \widehat{\pi}_{t-1}). \tag{190}$$

Now simplify the notation in (189) to:

$$0 = E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} \left\{ \widetilde{p}_t \ \Omega_{t+j} - s_\theta mc_{t+j} \right\}, \tag{191}$$

by defining:

$$\begin{aligned} \Gamma_{t+j} &\equiv \overline{\Lambda}^{\circ}_{t+j} q_{t+j} (\pi_{t+j})^{\theta}, \qquad \Omega_{t+j} \equiv \frac{\pi_t}{\pi_{t+j}}, \\ s_\theta &\equiv \frac{\theta}{\theta - 1}, \qquad \gamma \equiv \beta\alpha. \end{aligned}$$

Now rewrite (191) as:

$$\widetilde{p}_t E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} \ \Omega_{t+j} = s_\theta E_t \sum_{j=0}^{\infty} \gamma^j \Gamma_{t+j} mc_{t+j}. \tag{192}$$

Recall that the steady state value of $\widetilde{p}_t$ is equal to 1, as is that of Ω_{t+j} by construction. Then, since $\gamma < 1$, the steady state for (192) is:

$$\Gamma \sum_{j=0}^{\infty} \gamma^j = \frac{\Gamma}{1-\gamma} = s_\theta \frac{\Gamma}{1-\gamma} mc = s_\theta \Gamma mc \sum_{j=0}^{\infty} \gamma^j. \tag{193}$$

Dividing term by term (192) by (193), and taking logs, yields:

$$\begin{aligned} &\widehat{\widetilde{p}}_t + \log \left((1-\gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\log \frac{\Gamma_{t+j}}{\Gamma} \right) \exp \left(\log \Omega_{t+j} \right) \right) \\ = \ & \log \left((1-\gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp \left(\log \frac{\Gamma_{t+j}}{\Gamma} \right) \exp \left(\log \frac{mc_{t+j}}{mc} \right) \right). \end{aligned}$$

We rewrite this as:

$$\widehat{\widetilde{p}}_t + H(\widehat{\Gamma}_t, \widehat{\Omega}_t, \widehat{\Gamma}_{t+1}, \widehat{\Omega}_{t+1}, ...) = J(\widehat{\Gamma}_t, \widehat{mc}_t, \widehat{\Gamma}_{t+1}, \widehat{mc}_{t+1}, ...). \tag{194}$$

where

$$H(\widehat{\Gamma}_t, \widehat{\Omega}_t, \widehat{\Gamma}_{t+1}, \widehat{\Omega}_{t+1}, ...) \equiv \log\left((1-\gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp\left(\widehat{\Gamma}_{t+j}\right) \exp\left(\widehat{\Omega}_{t+j}\right)\right)$$

$$J(\widehat{\Gamma}_t, \widehat{mc}_t, \widehat{\Gamma}_{t+1}, \widehat{mc}_{t+1}, ...) \equiv \log\left((1-\gamma) E_t \sum_{j=0}^{\infty} \gamma^j \exp\left(\widehat{\Gamma}_{t+j}\right) \exp\left(\widehat{mc}_{t+j}\right)\right).$$

Now we log-linearize H and J, as we did above for G, noting that 1) $\Omega_t \equiv 1$, so the corresponding term disappears, and 2) the partial derivatives of H and J with respect to $\widehat{\Gamma}_{t+j}$ are the same, so that the corresponding terms cancel out in the linear approximation of (194). Hence, we are left with:

$$\begin{aligned} & \widehat{\widetilde{p}}_t + \gamma(1-\gamma) E_t\widehat{\Omega}_{t+1} + \gamma^2 (1-\gamma) E_t\widehat{\Omega}_{t+2} + ... \\ = \; & (1-\gamma)\widehat{mc}_t + \gamma(1-\gamma)\widehat{mc}_{t+1} + \gamma^2(1-\gamma)\widehat{mc}_{t+2} .... \end{aligned} \tag{195}$$

Using the definition of Ω_t, its log-linear deviation from steady state is:

$$\widehat{\Omega}_{t+j} \equiv \widehat{\pi}_t - \widehat{\pi}_{t+j},$$

so (195) becomes:

$$\begin{aligned} & \widehat{\widetilde{p}}_t + \gamma(1-\gamma) E_t(\widehat{\pi}_t - \widehat{\pi}_{t+1}) + \gamma^2(1-\gamma) E_t(\widehat{\pi}_t - \widehat{\pi}_{t+2}) + ... \\ = \; & (1-\gamma)\widehat{mc}_t + \gamma(1-\gamma)\widehat{mc}_{t+1} + \gamma^2(1-\gamma)\widehat{mc}_{t+2} .... \end{aligned}$$

which can be rearranged to:

$$\widehat{\widetilde{p}}_t + \widehat{\pi}_t = (1-\gamma)\sum_{j=0}^{\infty}\gamma^j E_t\left(\widehat{mc}_{t+j} + \widehat{\pi}_{t+j}\right).$$

Now, notice that this implies:

$$\widehat{\widetilde{p}}_t + \widehat{\pi}_t = (1-\gamma)\left(\widehat{mc}_t + \widehat{\pi}_t\right) + \gamma E_t\left(\widehat{\widetilde{p}}_{t+1} + \widehat{\pi}_{t+1}\right),$$

and hence (replacing γ by its original expression):

$$\widehat{\widetilde{p}}_t = (1-\beta\alpha)\widehat{mc}_t + \beta\alpha E_t\left(\widehat{\widetilde{p}}_{t+1} + \widehat{\pi}_{t+1} - \widehat{\pi}_t\right).$$

Now we use (190) to eliminate $\widehat{\widetilde{p}}_t$ and $\widehat{\widetilde{p}}_{t+1}$, and finally obtain the log-linearized Phillips equation:

$$\widehat{\pi}_t - \widehat{\pi}_{t-1} = \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\widehat{mc}_t + \beta E_t\left(\widehat{\pi}_{t+1} - \widehat{\pi}_t\right).$$

Phillips equation for wages

In this case the equations are:

$$\left(\pi_t^w\right)^{1-\theta} = \alpha_W \left(\pi_{t-1}^w\right)^{1-\theta} + \left(1-\alpha_W\right)\left(\widetilde{w}_t \pi_t^w\right)^{1-\theta}, \tag{196}$$

$$\begin{aligned} 0 &= E_t \sum_{j=0}^{\infty} \left(\beta \alpha_W\right)^j \lambda_{t+j}^{\circ} h_{t+j} \overline{w}_{t+j} \left(\pi_{t+j}^w\right)^{\psi} \\ &\quad \left\{ \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w}\right) - \frac{\psi}{\psi-1} \frac{\eta_H z_{t+j}^H h_{t+j}^{\chi}}{\lambda_{t+j}^{\circ} \overline{w}_{t+j}} \left(\frac{\widetilde{w}_t \pi_t^w}{\pi_{t+j}^w}\right)^{-\psi\chi} \right\}. \end{aligned} \tag{197}$$

Repeating the procedure used for (188), the log-linear version of (196) is:

$$\widehat{\widetilde{w}}_t = \frac{\alpha_W}{1-\alpha_W}\left(\widehat{\pi}_t^w - \widehat{\pi}_{t-1}^w\right). \tag{198}$$

Now divide through (197) by $\left(\widetilde{w}_t \pi_t^w\right)^{-\psi\chi}$ and simplify the notation to:

$$0 = E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w \left\{ \left(\widetilde{w}_t \pi_t^w\right)^{1+\psi\chi} - s_w \Psi_{t+j}^w \left(\pi_{t+j}^w\right)^{1+\psi\chi} \right\},$$

by defining:

$$\begin{aligned} \Gamma_{t+j}^w &\equiv \lambda_{t+j}^{\circ} h_{t+j} \overline{w}_{t+j} \left(\pi_{t+j}^w\right)^{\psi-1}, & \Psi_{t+j}^w &\equiv \frac{\eta_H z_{t+j}^H h_{t+j}^{\chi}}{\lambda_{t+j}^{\circ} \overline{w}_{t+j}}, \\ s_w &\equiv \frac{\psi}{\psi-1}, & \gamma_W &\equiv \beta \alpha_W. \end{aligned}$$

and rewrite it as:

$$\left(\widetilde{w}_t \pi_t^w\right)^{1+\psi\chi} E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w = s_w E_t \sum_{j=0}^{\infty} \gamma_W^j \Gamma_{t+j}^w \Psi_{t+j}^w \left(\pi_{t+j}^w\right)^{1+\psi\chi}, \tag{199}$$

The steady state value of $\widetilde{w}_t$ is 1, so the steady state for (199) is:

$$\pi^w \frac{\Gamma^w}{1-\gamma_W} = s_w \frac{\Gamma^w}{1-\gamma_W} \Psi^w \left(\pi^w\right)^{1+\psi\chi}. \tag{200}$$

Dividing term by term, the last two equations yields:

$$\left(1+\psi\chi\right)\left(\widehat{\widetilde{w}}_t + \widehat{\pi}_t^w\right) + H^w(\widehat{\Gamma}_t^w, \widehat{\Gamma}_{t+1}^w, \ldots) = J^w(\widehat{\Gamma}_t^w, \widehat{\Psi}_t^w, \widehat{\Gamma}_{t+1}^w, \widehat{\Psi}_{t+1}^w, \ldots), \tag{201}$$

where

$$H^w(\widehat{\Gamma}_t^w, \widehat{\Gamma}_{t+1}^w, \ldots) \equiv \log\left(\left(1-\gamma_W\right) E_t \sum_{j=0}^{\infty} \gamma_W^j \exp\left(\widehat{\Gamma}_{t+j}^w\right) \right)$$

$$\begin{aligned} & J^w(\widehat{\Gamma}_t^w, \widehat{\Psi}_t^w, \widehat{\pi}_t^w, \widehat{\Gamma}_{t+1}^w, \widehat{\Psi}_{t+1}^w, \widehat{\pi}_{t+1}^w, \ldots) \\ \equiv\ & \log\left(\left(1-\gamma_W\right) E_t \sum_{j=0}^{\infty} \gamma_W^j \exp\left(\widehat{\Gamma}_{t+j}^w\right) \exp\left(\widehat{\Psi}_{t+j}^w\right) \exp\left(\left(1+\psi\chi\right)\widehat{\pi}_{t+j}^w\right) \right). \end{aligned}$$

As above, we log-linearize H^w and J^w, noting that the partial derivatives of H^w and J^w with respect to $\widehat{\Gamma}_{t+j}$ are the same and cancel out in the linear approximation to (201). We obtain:

$$(1+\psi\chi)\left(\widehat{\widetilde{w}}_t+\widehat{\pi}_t^w\right)=(1-\gamma^w)\sum_{j=0}^{\infty}\gamma_W^j E_t\left(\widehat{\Psi}_{t+j}^w+(1+\psi\chi)\,\widehat{\pi}_{t+j}^w\right)$$

which implies:

$$\begin{aligned}(1+\psi\chi)\left(\widehat{\widetilde{w}}_t+\widehat{\pi}_t^w\right) &= (1-\gamma^w)\left(\widehat{\Psi}_t^w+(1+\psi\chi)\,\widehat{\pi}_t^w\right)\\ &\quad+(1+\psi\chi)\,\gamma^w E_t\left(\widehat{\widetilde{w}}_{t+1}+\widehat{\pi}_{t+1}^w\right),\end{aligned}$$

and hence:

$$\widehat{\widetilde{w}}_t=\frac{1-\beta\alpha_W}{1+\psi\chi}\widehat{\Psi}_t^w+\beta\alpha_W E_t\left(\widehat{\widetilde{w}}_{t+1}+\widehat{\pi}_{t+1}^w-\widehat{\pi}_t^w\right).$$

Now use (198) to eliminate $\widehat{\widetilde{w}}_t$ and $\widehat{\widetilde{w}}_{t+1}$:

$$\widehat{\pi}_t^w-\widehat{\pi}_{t-1}^w=\frac{(1-\alpha_W)\,(1-\beta\alpha_W)}{\alpha_W\,(1+\psi\chi)}\widehat{\Psi}_t^w+\beta E_t\left(\widehat{\pi}_{t+1}^w-\widehat{\pi}_t^w\right).$$

Finally, the definition of Ψ_t^w implies:

$$\widehat{\Psi}_t^w=\chi\widehat{h}_t+\widehat{z}_t^H-\widehat{\lambda}_t^\circ-\widehat{w}_t,$$

so substituting in the last expression yields the log-linearized Phillips equation for wages:

$$\widehat{\pi}_t^w-\widehat{\pi}_{t-1}^w=\frac{(1-\alpha_W)\,(1-\beta\alpha_W)}{\alpha_W\,(1+\psi\chi)}\left(\chi\widehat{h}_t+\widehat{z}_t^H-\widehat{\lambda}_t^\circ-\widehat{w}_t\right)+\beta E_t\left(\widehat{\pi}_{t+1}^w-\widehat{\pi}_t^w\right).$$

Appendix 2: Calibrated parameters and great ratios

The non-policy primitive parameters involved in the log-linearized structural equations are in Table 1. We use different values for the policy parameters in the simple feedback rules h_0, h_1, h_2, h_3, h_4, and k_0, k_1.

Table 1

PARAMETERS AND GREAT RATIOS

HOUSEHOLDS		
Intertemporal discount factor	β	0.996006
Habit parameter	ξ	0.75
Physical capital depreciation	δ^K	0.03940399
Labor supply elasticity	χ	1
DOMESTIC SECTOR FIRMS		
Fraction of rental bill that is bank financed	ς^K	0.19991058
Fraction of wage bill that is bank financed	ς^W	0.19991058
Fraction of primary inputs bill that is bank financed	ς^A	0.19991058
Fraction of imported inputs bill that is bank financed	ς^N	0.19991058
Production function parameter	a^q	0.25248933
Production function parameter	b^q	0.59297205
Production function parameter	c^q	0.037131742
Production function fixed cost	F^D	0
PRIMARY SECTOR FIRMS		
Production function coefficient	α^A	0.37142857
Production function coefficient	α^B	0.25032794
BANKS		
Fraction with rational expectations	β^B	0.5
Steady state cash demand as fraction of deposits	γ^B	0.06
Steady state regulatory reserves as fraction of deposits	γ^R	0.06
Foreign currency cash demand as fraction of foreign debt	γ^{FX}	0.067
Cost function parameter	a_0^B	1.1
Cost function parameter	a_L^B	1.1039989
Cost function parameter	a_D^B	1.0964033
Foreign debt steady state exogenous risk premium	ϕ^{**B}	0.006
Foreign debt endogenous risk premium parameter	α_1^{RP}	4.0023129×10^{-6}
Foreign debt endogenous risk premium parameter	α_2^{RP}	1.6666667

Table 1 (continued)

PROBABILITY OF NOT OPTIMIZING WAGES/PRICES		
Wages	α_W	0.6
Domestic goods	α	0.5
Imported goods	α_N	0.55
Exported manufactured goods	α_X	0.35
AUXILIARY COST FUNCTIONS		
Physical capital utilization intensity cost function parameter	a_u	100
Investment adjustment cost function parameter	a_V,	3.6
Transactions cost function parameter	a_M	7.1746188
Transactions cost function parameter	b_M	0.28374755
Transactions cost function parameter	c_M	2.6115714
LRW		
LRW steady state growth	μ^{z**}	1.04
LRW risk free interest rate	i^{**}	0.05369
Inflation	π^{**}	1.023
MONETARY POLICY		
Target inflation rate	π^T	1.05
Target SS Int. Reserves/Financial System liabilities	γ^T	0.36743923
OTHER STEADY STATE RATES		
Steady state Central Bank bond interest rate	i	0.065922041
Steady state currency depreciation rate	δ	1.0263930
Steady state wage inflation rate	π^w	1.092
Steady state deposit rate	i^D	0.0963789
Steady state loan rate	i^L	0.15
Steady state rental rate	i^K	0.084997896
Real marginal cost	mc	0.79839678

Table 1 (continued)

FISCAL		
Government expenditures/GDP	g/y	0.22
Lump sum taxes/GDP	t/y	0.244
Steady state bank loans to Government/GDP	ℓ^G/y	0.04
OTHER STEADY STATE RATIOS		
Consumption/GDP	$p^C c/y$	0.65
Investment/GDP	$p^V v/y$	0.19889
Exports/GDP	x/y	0.24111
Primary sector exports/Exports	$ep^{**A}x^A/x$	0.30564
Primary sector domestic sector inputs/GDP	$\alpha_A p^A a/y$	0.039
Domestic sector primary sector inputs/GDP	$p^A q^{AD}/y$	0.031309858
Domestic sector domestic intermediate output/GDP	q^{ID}/y	0.0624889
Domestic sector imported intermediate output/GDP	q^{IN}/y	0.099
Domestic sector output/GDP	q/y	1.087798 8
Primary sector output/GDP	$p^A a/y$	0.105

Appendix 3 Definitions of the coefficients in the log-linearized equations and their calibrated values

The following compound parameters have been used in the log-linearization of the systems:

Structure of bank deposit supply:

$$\alpha_{DS}^{B} \equiv \frac{a^{B}d}{a^{B}d + a_{L}^{B}\left[i^{D} - (1-\gamma^{B}-\gamma^{R})i\right]} = 0.541\,982\,09$$

Structure of bank deposit margin:

$$\begin{aligned}
\alpha_{1}^{MD} &\equiv \frac{1+i^{D}}{i^{D} - (1-\gamma^{B}-\gamma^{R})i} = 28.\,575\,716 \\
\alpha_{2}^{MD} &\equiv \frac{(1-\gamma^{B}-\gamma^{R})\,(1+i)}{i^{D} - (1-\gamma^{B}-\gamma^{R})i} = 24.\,448\,069 \\
\alpha_{3}^{MD} &\equiv \frac{(1-\gamma^{B}-\gamma^{R})i}{i^{D} - (1-\gamma^{B}-\gamma^{R})i} = 1.\,511\,993\,0
\end{aligned}$$

Structure of bank lending margin:

$$\alpha^{ML} \equiv \frac{1+i}{i^{L}-i} = 12.\,677\,782$$

Structure of deposit drains:

$$\begin{aligned}
\alpha_{B}^{MD} &\equiv \frac{\gamma^{B}}{1-\gamma^{B}-\gamma^{R}} = 6.\,818\,181\,8\times 10^{-2} \\
\alpha_{R}^{MD} &\equiv \frac{\gamma^{R}}{1-\gamma^{B}-\gamma^{R}} = 6.\,818\,181\,8\times 10^{-2}
\end{aligned}$$

Structure of Bank assets:

$$\alpha_{A}^{B} \equiv \frac{\ell}{\ell + b^{CB}} = 0.860\,227\,97$$

Structure of Bank liabilities:

$$\alpha_{L}^{B} \equiv \frac{d(1-\gamma^{B}-\gamma^{R})}{d(1-\gamma^{B}-\gamma^{R}) + eb^{*B}} = 0.746\,376\,18$$

Structure of cash demand:

$$a^{CM} \equiv \frac{\varpi\left[p^{C}c + p^{V}v\right]}{\varpi\left[p^{C}c + p^{V}v\right] + \gamma^{B}d} = 0.837\,260\,45$$

Structure of private absorption:

$$\alpha_{A}^{CM} \equiv \frac{p^{C}c}{p^{C}c + p^{V}v} = 0.765\,705\,8$$

Elasticity of household cash-absorption ratio ϖ w.r. to the gross deposit interest rate:

$$\alpha_{D}^{CM} \equiv \frac{1}{(b_{M}+1)\left[(a_{M}+1)\left(1+i^{D}\right)-1\right]} = 0.09783$$

Elasticity of household cash-absorption ratio ϖ w.r. to a (positive) household cash demand shock:

$$\alpha_M^{CM} \equiv \frac{a_M\left(1+i^D\right)}{a_M\left(1+i^D\right)+i^D} = 0.98789587$$

Elasticity of auxiliary transactions cost function $\widetilde{\tau}_M$ w.r. to the gross deposit interest rate:

$$b_\tau^Q \equiv \frac{b_M\varpi^{-b_M} - a_M\varpi}{a_M\varpi + \varpi^{-b_M} - c_M}\alpha_D^{CM} = 0.068799004$$

Elasticity of auxiliary function $\widetilde{\varphi}_M$ w.r. to the gross domestic interest rate:

$$\varepsilon_M = \frac{b_M\left(1+b_M\right)\varpi^{-b_M}}{1-c_M+\left(1+b_M\right)\varpi^{-b_M}}\alpha_D^{CM} = 0.071745469$$

Structure of aggregate supply:

$$\begin{aligned}
a^Y &\equiv \frac{y}{y+p^N n} = 0.81967213, \\
\alpha_c^Y &\equiv \frac{p^C c}{p^C c + p^V v + x^M + ep^{**A}x^A + g} = 0.532\,786\,88 \\
\alpha_v^Y &\equiv \frac{p^V v}{p^C c + p^V v + x^M + ep^{**A}x^A + g} = 0.163\,024\,59 \\
\alpha_{xm}^Y &\equiv \frac{x^M}{p^C c + p^V v + x^M + ep^{**A}x^A + g} = 0.137\,227\,16 \\
\alpha_{xa}^Y &\equiv \frac{ep^{**A}x^A}{p^C c + p^V v + x^M + ep^{**A}x^A + g} = 0.060403984
\end{aligned}$$

Elasticities of primary goods supply:

$$\begin{aligned}
\gamma_1^a &\equiv \frac{\alpha_A + \beta_A}{1-\alpha_A-\beta_A} = 1.\,643\,799\,6 \\
\gamma_2^a &\equiv \frac{\beta_A}{1-\alpha_A-\beta_A} = 0.661\,816\,93
\end{aligned}$$

Elasticity of intermediate domestic demand w.r. to the rental rate:

$$a_0^{qD} \equiv \frac{ki^K}{a_u q^{ID}} = 0.035433109$$

Structure of intermediate domestic demand:

$$\begin{aligned}
a_1^{qD} &\equiv \frac{\widetilde{\tau}_M\left[p^C c + p^V v\right]}{\widetilde{\tau}_M\left[p^C c + p^V v\right] + C^B + \alpha_A e p^{**A} a} = 0.135\,850\,44 \\
a_2^{qD} &\equiv \frac{C^B}{\widetilde{\tau}_M\left[p^C c + p^V v\right] + C^B + \alpha_A e p^{**A} a} = 0.240\,042\,63
\end{aligned}$$

Structure of loan supply:

$$\alpha_{LS}^B \equiv \frac{a^B\ell}{a^B\ell + a_0^B\left[i^D - (1-\gamma^B-\gamma^R)i\right]} = 0.542\,144\,06$$

Structure of Central Bank liabilities:

$$a_1^{CB} \equiv \frac{m^0}{m^0+\gamma^R d+b^{CB}} = 0.582\,191\,78$$
$$a_2^{CB} \equiv \frac{\gamma^R d}{m^0+\gamma^R d+b^{CB}} = 0.113\,013\,70$$

Structure of bank cost:

$$a^{BC} \equiv \frac{\ell\left(a_L^B \ell - a_0^B d\right)}{a_L^B \ell^2 + a_D^B d^2 - 2a_0^B \ell d} = 1.\,844\,117\,2$$

Structure of consumption price index:

$$a_{PC} \equiv \frac{(1-a_D)\left(p^N\right)^{1-\theta_C}}{a_D + (1-a_D)\left(p^N\right)^{1-\theta_C}} = 0.1$$

Structure of investment price index:

$$a_{PV} \equiv \frac{(1-b_D)\left(p^N\right)^{1-\theta_V}}{b_D + (1-b_D)\left(p^N\right)^{1-\theta_V}} = 0.2$$

Structure of Central Bank quasi-fiscal surplus:

$$a^{QF} = \frac{\left(1+i_t^{**}-\frac{1}{\delta}\right)\frac{er^{*CB}}{\pi^{**N}\mu^z}}{\left(1+i_t^{**}-\frac{1}{\delta}\right)\frac{er^{*CB}}{\pi^{**N}\mu^z} - ib^{CB}}$$
$$\alpha_i^{QF} = \frac{1+i^{**}}{1+i^{**}-\frac{1}{\delta}}$$

Structure of Government domestic surplus:

$$a_1^{GD} \equiv \frac{t}{t-g+\ell^G-\frac{1+i^L}{\mu^z\pi}\ell^G}$$
$$a_2^{GD} \equiv \frac{g}{t-g+\ell^G-\frac{1+i^L}{\mu^z\pi}\ell^G}$$
$$a_3^{GD} \equiv \frac{\ell^G}{t-g+\ell^G-\frac{1+i^L}{\mu^z\pi}\ell^G}$$

Structure of uses of balance of payments and fiscal resources:

$$a_1^{BP} \equiv \frac{r^{*CB}}{r^{*CB}+n+(1+i^{**})\left[1+\phi^{*B}+\alpha_1^{BP}\left(b^{*B}e\right)^{\alpha_2^{BP}}\right]\frac{b^{*B}}{\mu^{z**}\pi^{**N}}+\left(t-g-\frac{1+i^L}{\mu^{z**}\pi^{**N}}\ell^G\right)/e} = 0.298\,400\,53$$
$$a_2^{BP} \equiv \frac{n}{r^{*CB}+n+(1+i^{**})\left[1+\phi^{*B}+\alpha_1^{BP}\left(b^{*B}e\right)^{\alpha_2^{BP}}\right]\frac{b^{*B}}{\mu^{z**}\pi^{**N}}+\left(t-g-\frac{1+i^L}{\mu^{z**}\pi^{**N}}\ell^G\right)/e} = 0.504\,985\,51$$
$$a_3^{BP} \equiv \frac{(1+i^{**})\left[1+\phi^{*B}+\alpha_1^{BP}\left(b^{*B}e\right)^{\alpha_2^{BP}}\right]\frac{b^{*B}}{\mu^{z**}\pi^{**N}}}{r^{*CB}+n+(1+i^{**})\left[1+\phi^{*B}+\alpha_1^{BP}\left(b^{*B}e\right)^{\alpha_2^{BP}}\right]\frac{b^{*B}}{\mu^{z**}\pi^{**N}}+\left(t-g-\frac{1+i^L}{\mu^{z**}\pi^{**N}}\ell^G\right)/e} = 0.150\,706\,18$$

Structure of sources of balance of payments and fiscal resources:

$$a_4^{BPf} \equiv \frac{b^{*B}}{b^{*B}+p^{*XM}x^M+p^{**A}x^A+(1+i^{**})\frac{r^{*CB}}{\mu^{z**}\pi^{**N}}} = 0.143\,261\,03$$

$$a_5^{BPF} \equiv \frac{p^{*XM}x^M}{b^{*B}+p^{*XM}x^M+p^{**A}x^A+(1+i^{**})\frac{r^{*CB}}{\mu^{z**}\pi^{**N}}} = 0.387\,802\,92$$

$$a_6^{BP} \equiv \frac{p^{**A}x^A}{b^{*B}+p^{*XM}x^M+p^{**A}x^A+(1+i^{**})\frac{r^{*CB}}{\mu^{z**}\pi^{**N}}} = 0.170\,701\,20$$

Structure of Trade balance:

$$a_1^{TB} \equiv \frac{p^{*MX}x^M}{p^{*MX}x^M+p^{**A}x^A-n}$$

$$a_2^{TB} \equiv \frac{p^{**A}x^A}{p^{*MX}x^M+p^{**A}x^A-n}$$

Structure of net primary surplus:VUELA

$$\alpha_1^{BP} \equiv \frac{t/y}{\left[t-g-\left(\frac{1+i^L}{\mu^{z**}\pi^T}-1\right)\ell^G\right]/y} = \frac{0.244}{0.02} = 12.2$$

$$\alpha_2^{BP} \equiv \frac{\ell^G/y}{\left[t-g-\left(\frac{1+i^L}{\mu^{z**}\pi^T}-1\right)\ell^G\right]/y} = \frac{0.044}{0.02} = 2.2$$

$$\alpha_3^{BP} \equiv \frac{1+i^L}{\mu^{z**}\pi^T}\alpha_2^{BP} = 2.316\,849\,8$$

Structure of the real marginal cost base:

$$\alpha_q \equiv \frac{q}{q+F^D} = 1 \qquad \left(F^D=0\right)$$

Structure of physical capital services demand:

$$\gamma^K \equiv \frac{a^q\frac{mc(q+F^D)}{1+\varsigma^K i^L}}{a^q\frac{mc(q+F^D)}{1+\varsigma^K i^L}+\beta_A ep^{**A}a} = 0.890\,108\,8$$

Structure of firm factor/loan demands

$$\alpha_K^{MC} = \alpha_W^{MC} = \alpha_A^{MC} = \alpha_N^{MC} \equiv \frac{i^L}{1/\varsigma+i^L} = 0.029\,113\,57$$

Structure of joint domestic and primary demand:

$$a_y^Q \equiv \frac{y}{y+q^{ID}+p^Nn^F} = 0.860\,963\,89$$

$$a_D^Q \equiv \frac{q^{ID}}{y+q^{ID}+q^{IN}} = 5.380\,068\,6\times 10^{-2}$$

Structure of joint domestic and primary supply:

$$\gamma^Q \equiv \frac{q}{q+ep^{**A}x^A} = 0.936\,555\,45$$

Elasticity of real marginal cost w.r. to the gross loan interest rate:

$$\begin{aligned}\alpha_L^{MC} &\equiv \left[a^q \alpha_K^{MC} + b^q \alpha_W^{MC} + c^q \alpha_A^{MC} + \left(1 - a^q - b^q - c^q\right) \alpha_N^{MC}\right] \frac{1 + i^L}{i^L} \\ &= (0.029\,113\,57)\,\frac{1.15}{0.15} = 0.223\,204\,04\end{aligned}$$

Structure of import demand:

$$\begin{aligned}a_1^N &\equiv \frac{\left(1 - a_D\right) p^C c}{\left(1 - a_D\right) p^C c + \left(1 - b_D\right) p^V v + \frac{1 - a^q - b^q - c^q}{1 + \varsigma^N i^L} mc\left[q + F^D\right]} = 0.33 \\ a_2^N &\equiv \frac{\left(1 - b_D\right) p^V v}{\left(1 - a_D\right) p^C c + \left(1 - b_D\right) p^V v + \frac{1 - a^q - b^q - c^q}{1 + \varsigma^N i^L} mc\left[q + F^D\right]} = 0.22\end{aligned}$$

Structure of primary goods demand:

$$\gamma^{XA} \equiv \frac{c^q \frac{mc\left[q + F^D\right]}{\left(1 + \varsigma^A i^L\right) ep^{**A}}}{c^q \frac{mc\left[q + F^D\right]}{\left(1 + \varsigma^A i^L\right) ep^{**A}} + x^A} = 0.298\,189\,13$$

Structure of marginal utility of consumption:

$$\begin{aligned}a_C &\equiv \frac{\beta \xi}{\mu^z - \beta \xi} = 2.\,549\,542\,6 \\ \alpha_C &\equiv \frac{\xi}{\mu^z - \xi} = 2.\,586\,206\,9\end{aligned}$$

Structure of physical capital formation:

$$a_K \equiv \frac{\left(1 - \delta^K\right) k}{\left(1 - \delta^K\right) k + v \mu^z} = \frac{1 - \delta^K}{\mu^{z**}} = 0.923\,650\,01$$

Structure of bank risk premium:

$$a^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + \alpha_1^{RP} \left(b^{*B} e\right)^{\alpha_2^{RP}}} = 0.998\,511\,17$$

Structure of bank arbitrage premium:

$$\overline{a}^{RP} \equiv \frac{1 + \phi^{*B}}{1 + \phi^{*B} + \left(1 + \alpha_2^{RP}\right) \alpha_1^{RP} \left(b^{*B} e\right)^{\alpha_2^{RP}}} = 0.996\,039\,6$$

Structure of gross interbank (cum Central Bank bond) interest rate:

$$\begin{aligned}\gamma_1^B &\equiv \frac{\delta \left\{\left(1 + i^{**}\right) \left[1 + \phi^{B*} + \left(\alpha_2^{RP} + 1\right) \alpha_1^{RP} \left(e b^{*B}\right)^{\alpha_2^{RP}}\right] - 1\right\}}{\delta \left\{\left(1 + i^{**}\right) \left[1 + \phi^{B*} + \left(\alpha_2^{RP} + 1\right) \alpha_1^{RP} \left(e b^{*B}\right)^{\alpha_2^{RP}}\right] - 1\right\} + 1} = 0.061845086 \\ \gamma_2^B &\equiv \frac{\left(1 + i^{**}\right) \left[1 + \phi^{B*} + \left(\alpha_2^{RP} + 1\right) \alpha_1^{RP} \left(e b^{*B}\right)^{\alpha_2^{RP}}\right]}{\left(1 + i^{**}\right) \left[1 + \phi^{B*} + \left(\alpha_2^{RP} + 1\right) \alpha_1^{RP} \left(e b^{*B}\right)^{\alpha_2^{RP}}\right] - 1} = 16.\,569\,8\end{aligned}$$

Structure of firm loan demand:

$$\begin{aligned}
\gamma_1^{LM} &\equiv \frac{\frac{a^q}{1/\varsigma^K+i^L}}{\frac{a^q}{1/\varsigma^K+i^L}+\frac{b^q}{1/\varsigma^W+i^L}+\frac{c^q}{1/\varsigma^A+i^L}+\frac{1-a^q-b^q-c^q}{1/\varsigma^N+i^L}} = a^q = 0.252\,489\,33 \\
\gamma_2^{LM} &\equiv \frac{\frac{b^q}{1/\varsigma^W+i^L}}{\frac{a^q}{1/\varsigma^K+i^L}+\frac{b^q}{1/\varsigma^W+i^L}+\frac{c^q}{1/\varsigma^A+i^L}+\frac{1-a^q-b^q-c^q}{1/\varsigma^N+i^L}} = b^q = 0.592\,972\,05 \\
\gamma_3^{LM} &\equiv \frac{\frac{c^q}{1/\varsigma^A+i^L}}{\frac{a^q}{1/\varsigma^K+i^L}+\frac{b^q}{1/\varsigma^W+i^L}+\frac{c^q}{1/\varsigma^A+i^L}+\frac{1-a^q-b^q-c^q}{1/\varsigma^N+i^L}} = c^q = 3.\,713\,174\,2\times 10^{-2} \\
\delta_K^{LM} &\equiv \gamma_1^{LM}\frac{\alpha_K^{MC}}{\varsigma^K i^L} = 0.245\,138\,46 \\
\delta_W^{LM} &\equiv \gamma_2^{LM}\frac{\alpha_W^{MC}}{\varsigma^W i^L} = 0.575\,738\,98 \\
\delta_A^{LM} &\equiv \gamma_3^{LM}\frac{\alpha_A^{MC}}{\varsigma^A i^L} = 3.\,605\,261\,2\times 10^{-2} \\
\delta_N^{LM} &\equiv \left(1-\gamma_1^{LM}-\gamma_2^{LM}-\gamma_3^{LM}\right)\frac{\alpha_N^{MC}}{\varsigma^N i^L} = 0.113\,996\,32
\end{aligned}$$

Elasticity of private loan demand w.r. to the gross loan interest rate:

$$\gamma^{LM} \equiv \left(\delta_K^{LM}\varsigma^K+\delta_W^{LM}\varsigma^W+\delta_A^{LM}\varsigma^A+\delta_N^{LM}\varsigma^N\right)\left(1+i^L\right) = 0.223\,162\,89$$

Structure of total loan demand:

$$a^{LM} \equiv \frac{f_L(1+i^L)mc(q+F^D)}{f_L(1+i^L)mc(q+F^D)+\ell^G} = \frac{\ell-\ell^G}{\ell} = 0.799\,402\,15$$

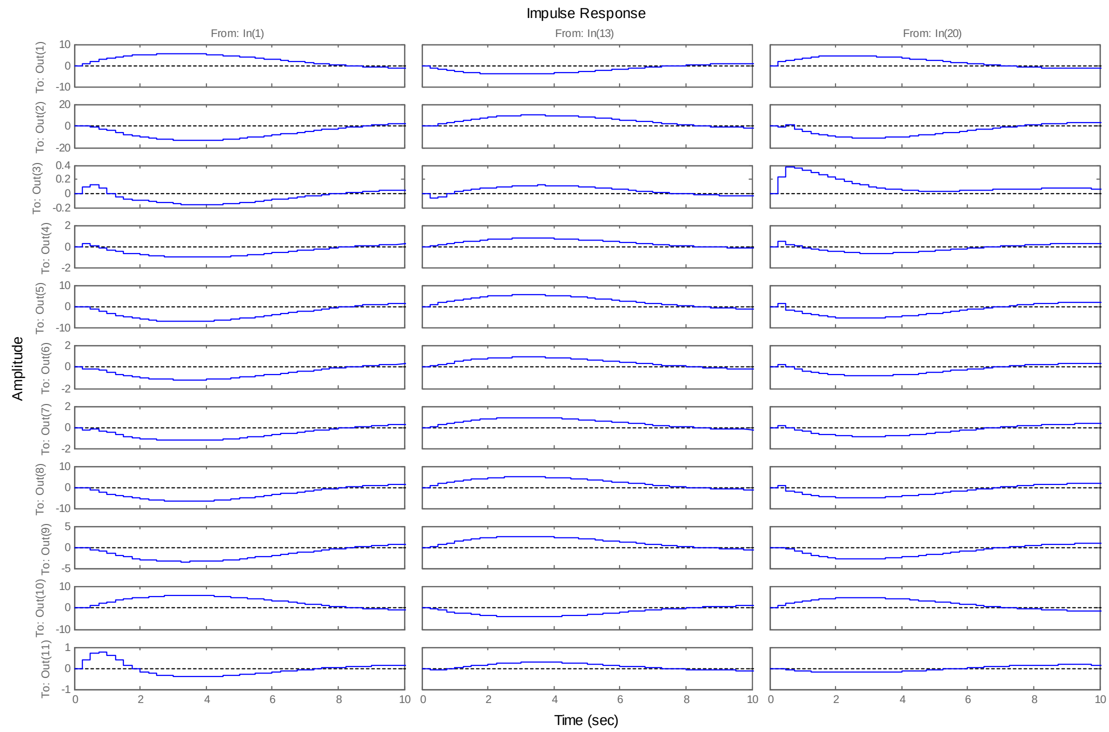
Appendix 4: Impulse Response Functions

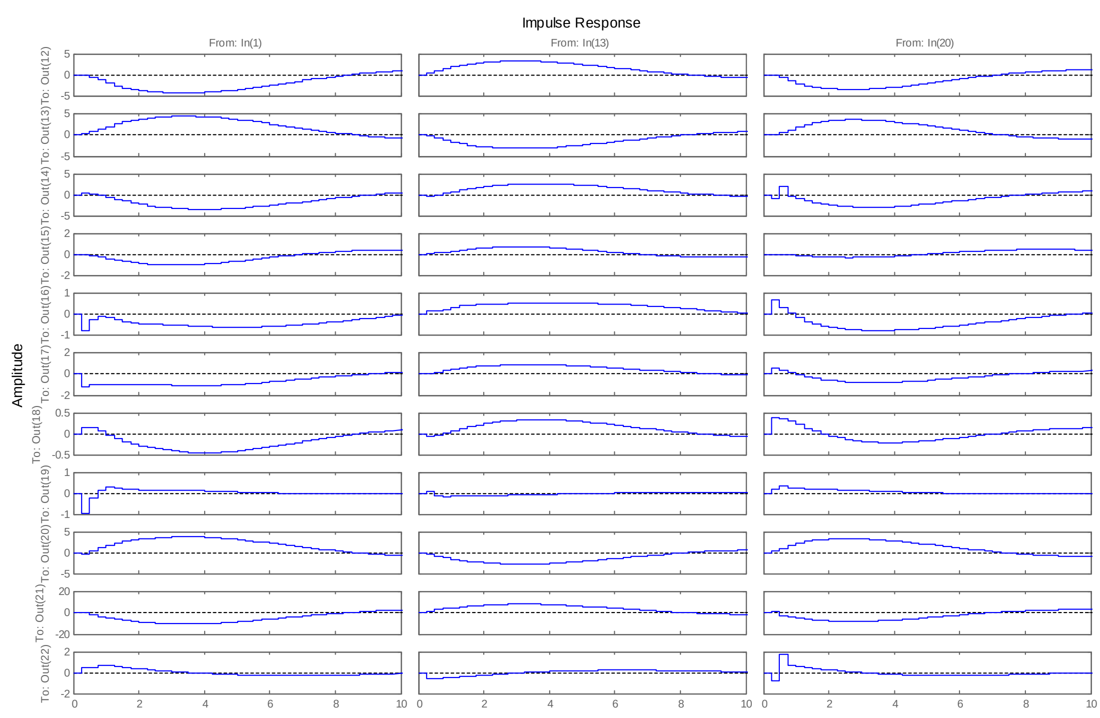
The following Impulse Response Functions have been plotted using the following coefficients on the simple policy rules:

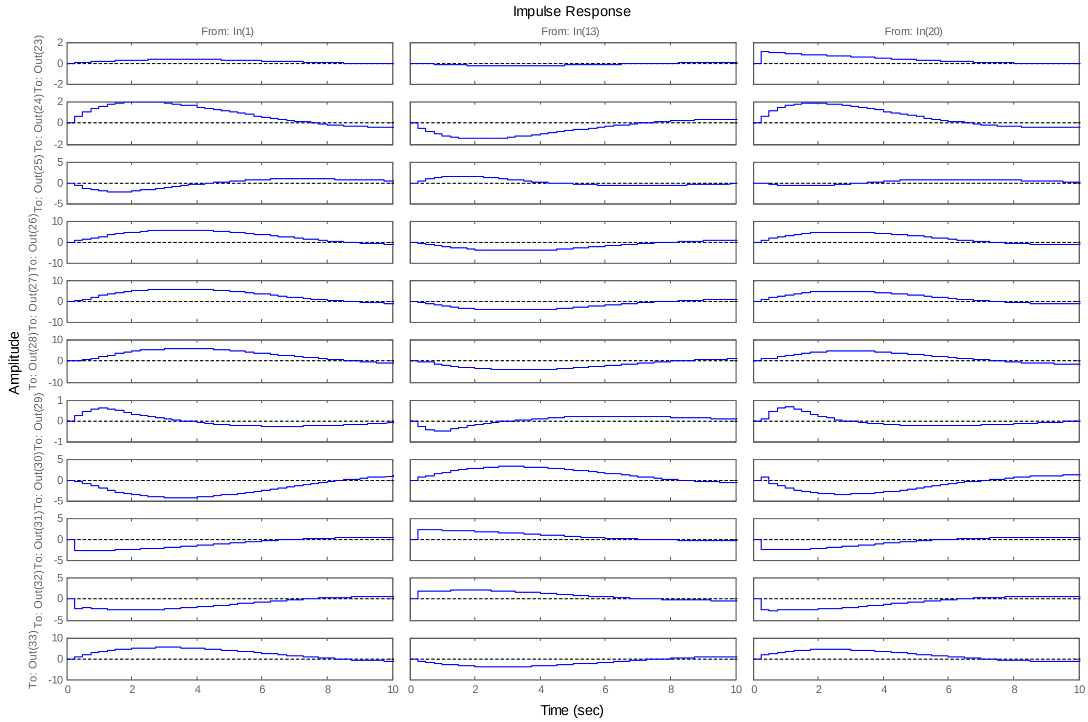
h_0	h_1	h_2	h_3	h_4	k_0	k_1
0.5	0.0	0.5	0.5	0.5	0.1	1.0

The time unit is a year, but the functions have been sampled every quarter. The impulses pertain to the exogenous variables shown above each set of three graphs, in the indicated order. The outputs correspond to the 33 elements of vector $\widetilde{d}_t$, in the same order. For convenience, we have separated them in three groups of 11.

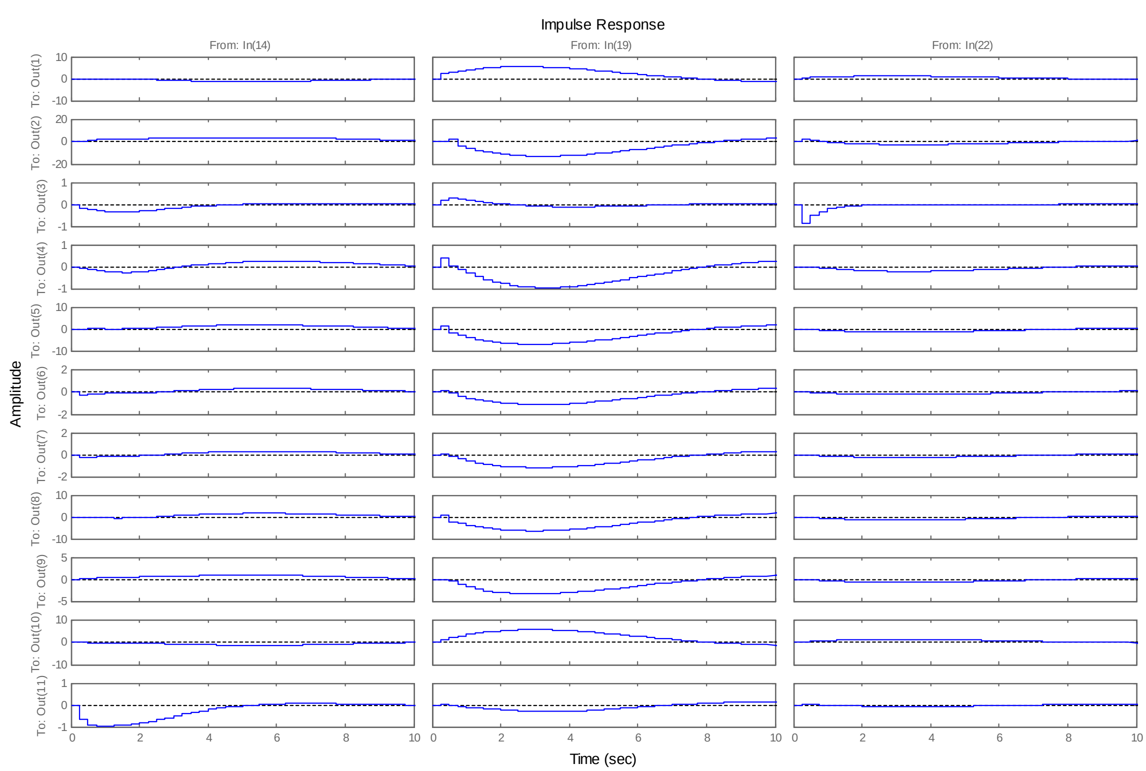
Responses to ϵ, t, and ϕ^{**B} :

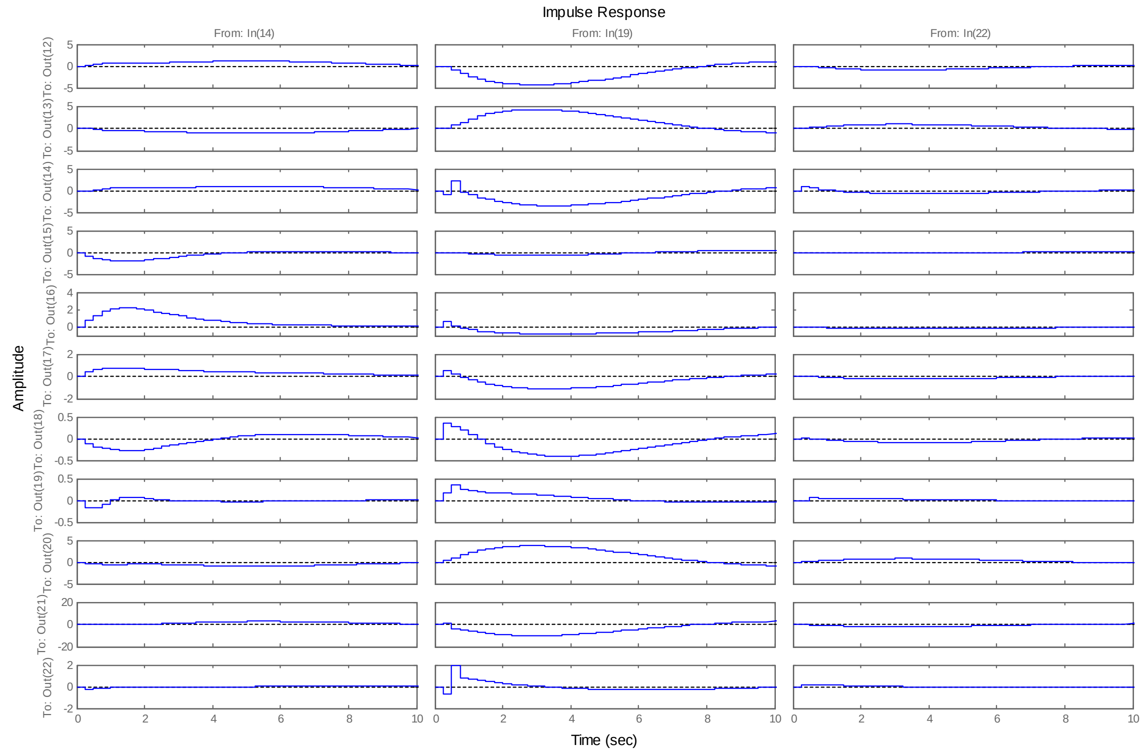


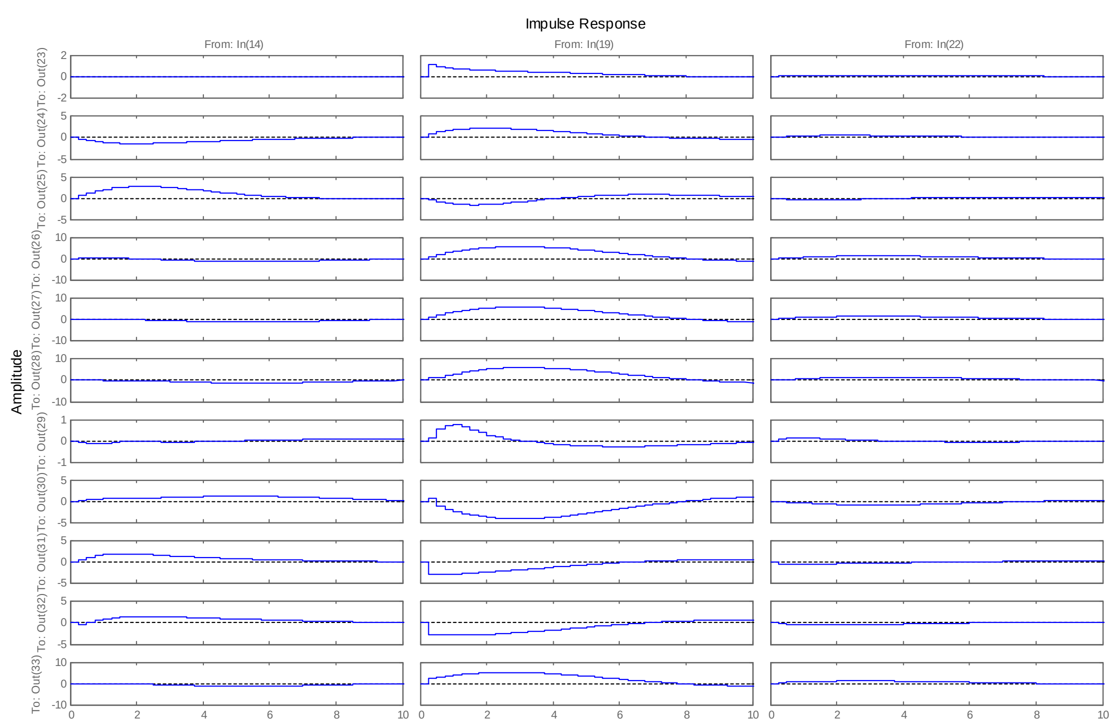




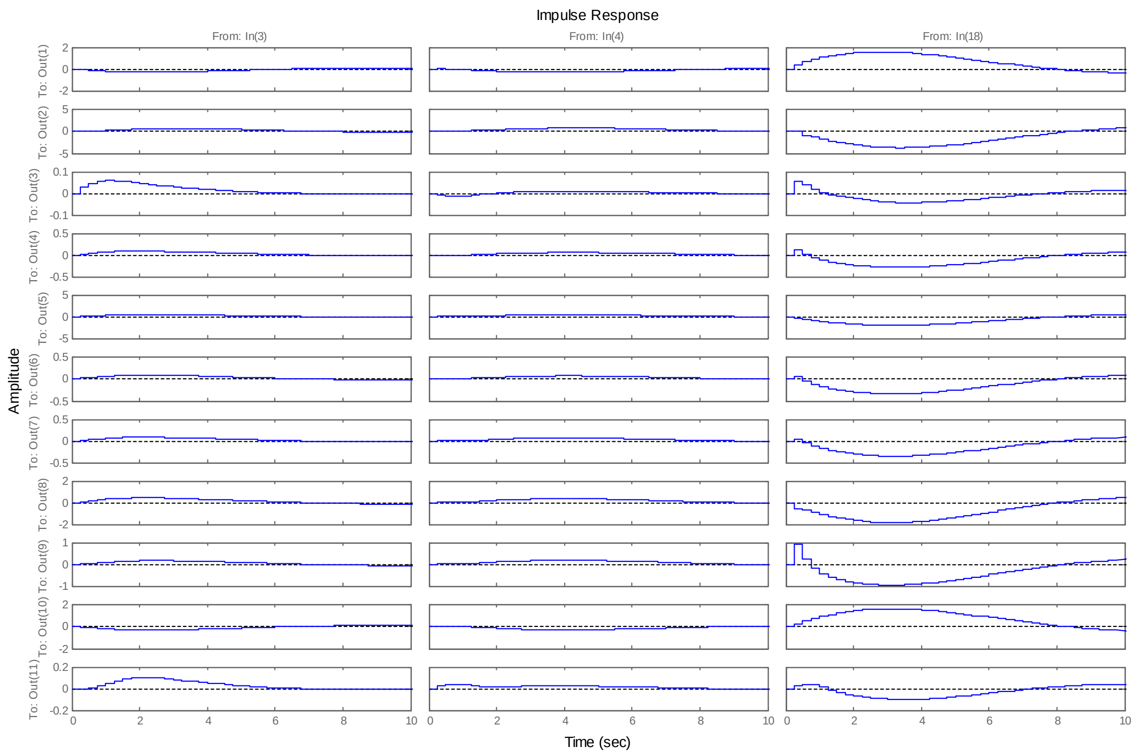
Responses to μ^{z**}, i^{**}, and z^M :

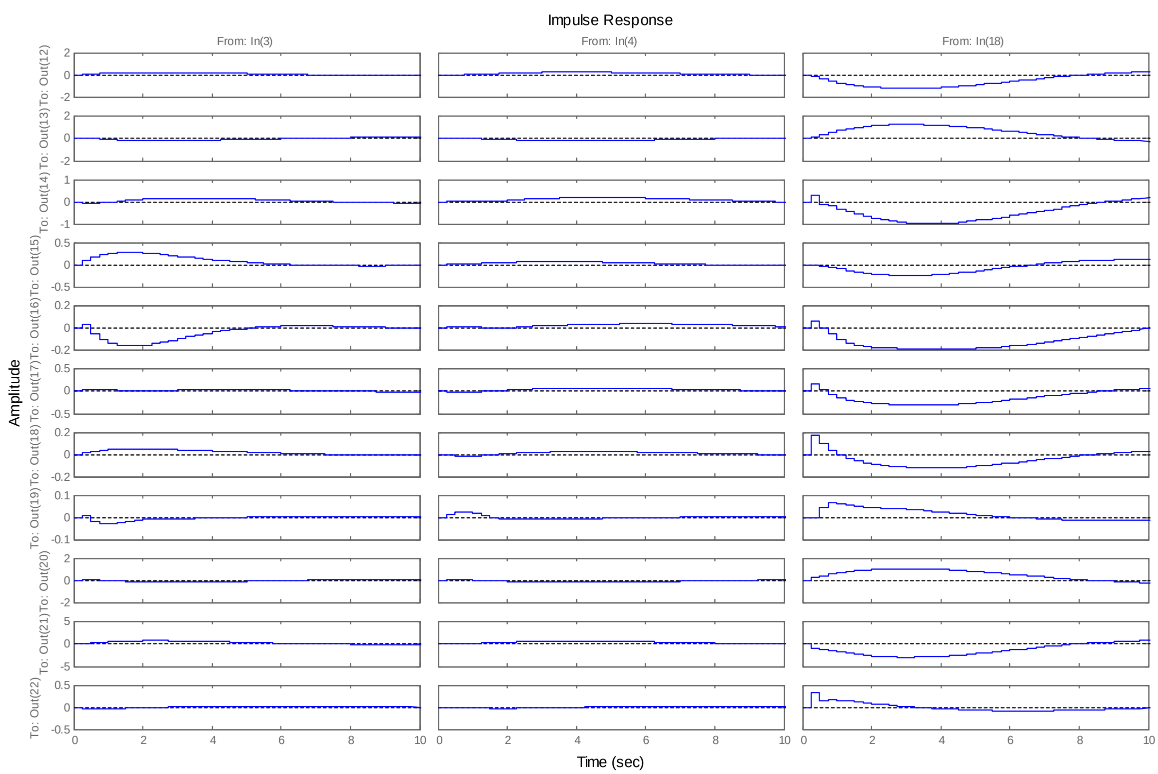


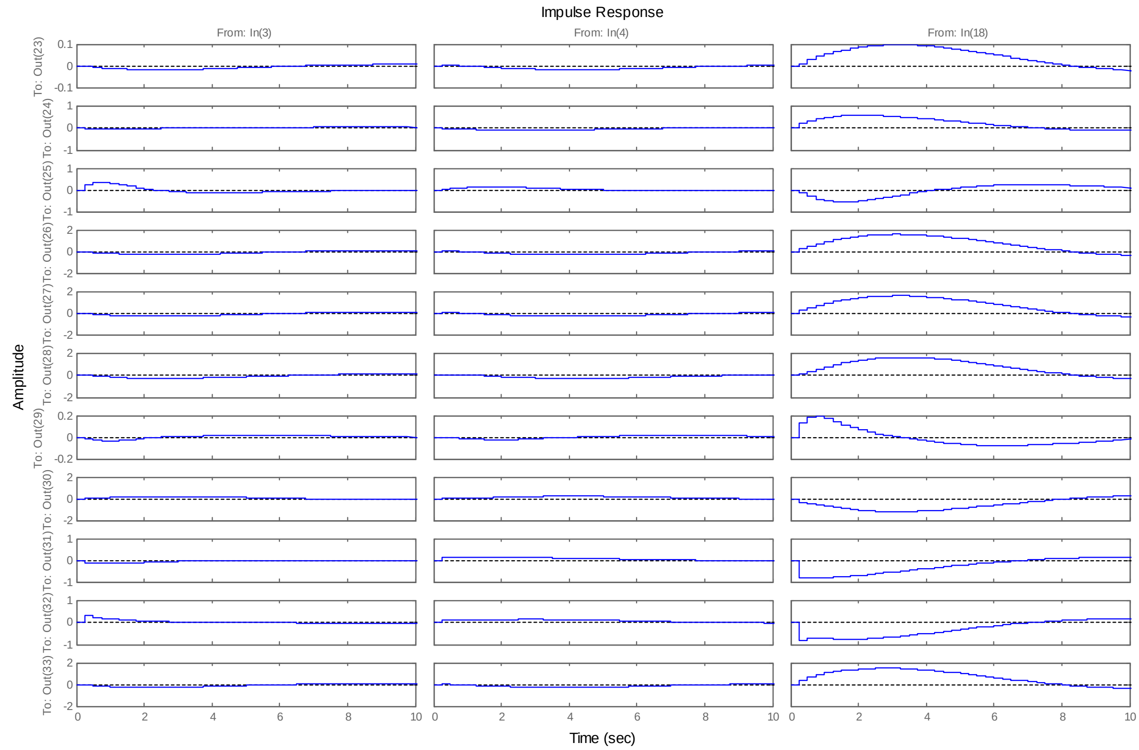

Impulse Response
From: In(14)
From: In(19)
From: In(22)
Amplitude
To: Out(12)
To: Out(13)
To: Out(14)
To: Out(15)
To: Out(16)
To: Out(17)
To: Out(18)
To: Out(19)
To: Out(20)
To: Out(21)
To: Out(22)
Time (sec)


Impulse Response
From: In(14)
From: In(19)
From: In(22)
Amplitude
To: Out(23)
To: Out(24)
To: Out(25)
To: Out(26)
To: Out(27)
To: Out(28)
To: Out(29)
To: Out(30)
To: Out(31)
To: Out(32)
To: Out(33)
Time (sec)

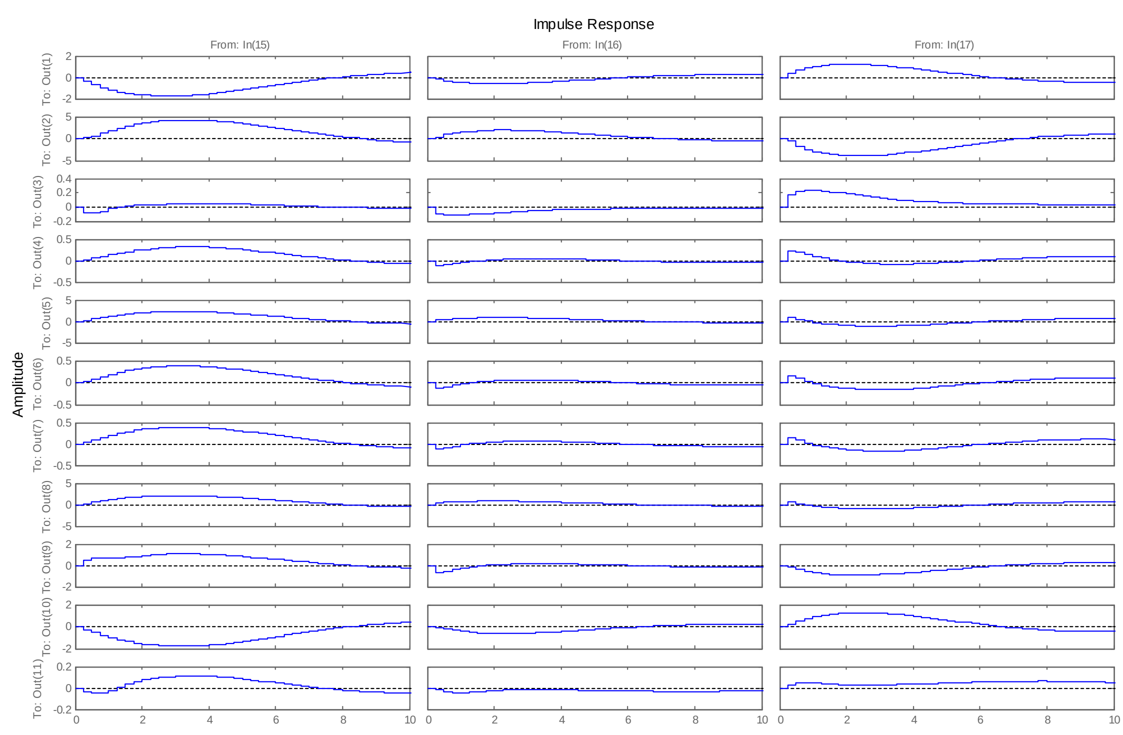
Responses to z^V, z^H, and y^{**} :

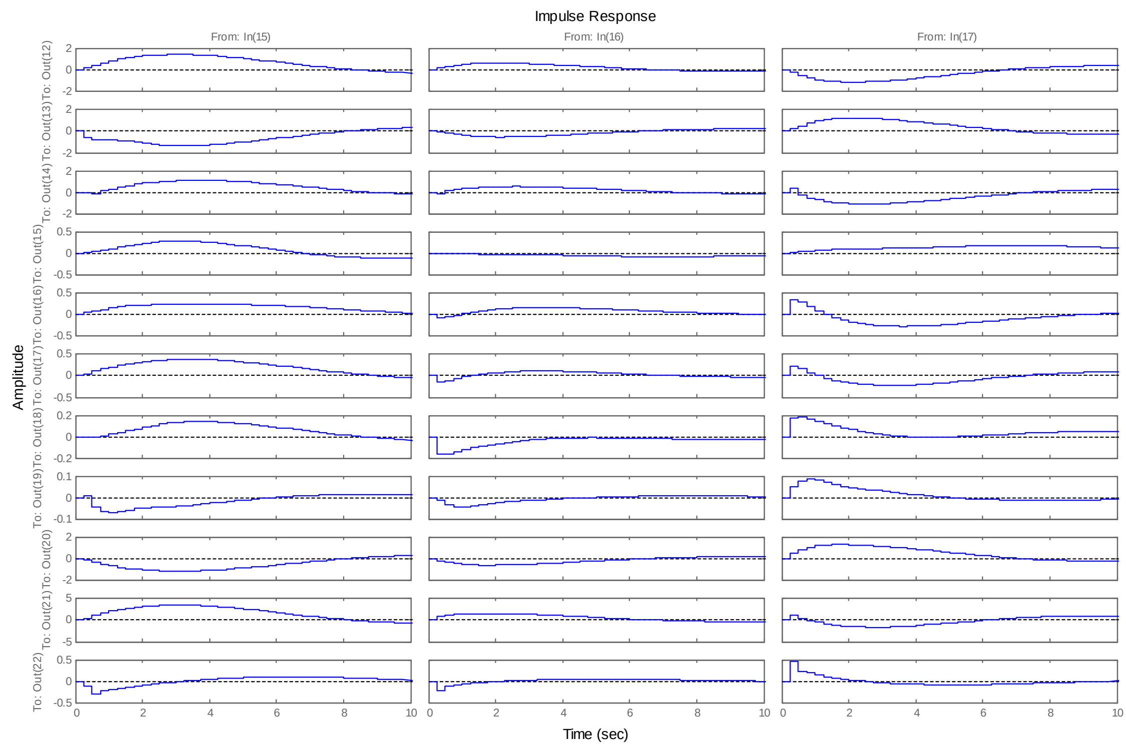


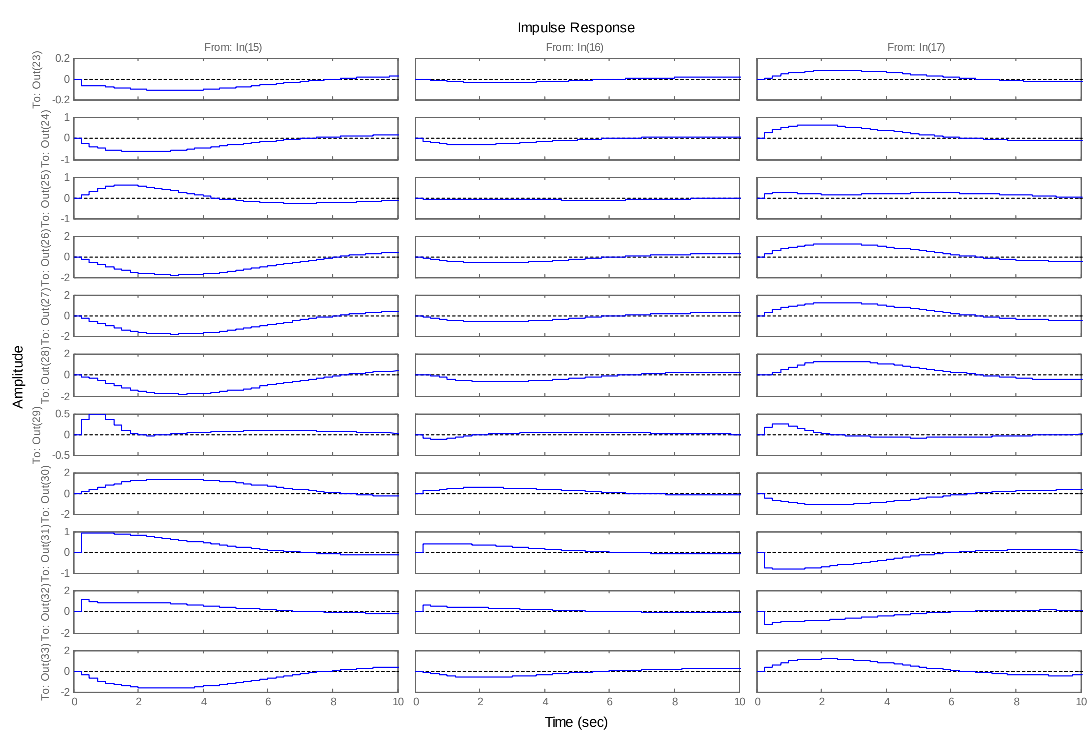




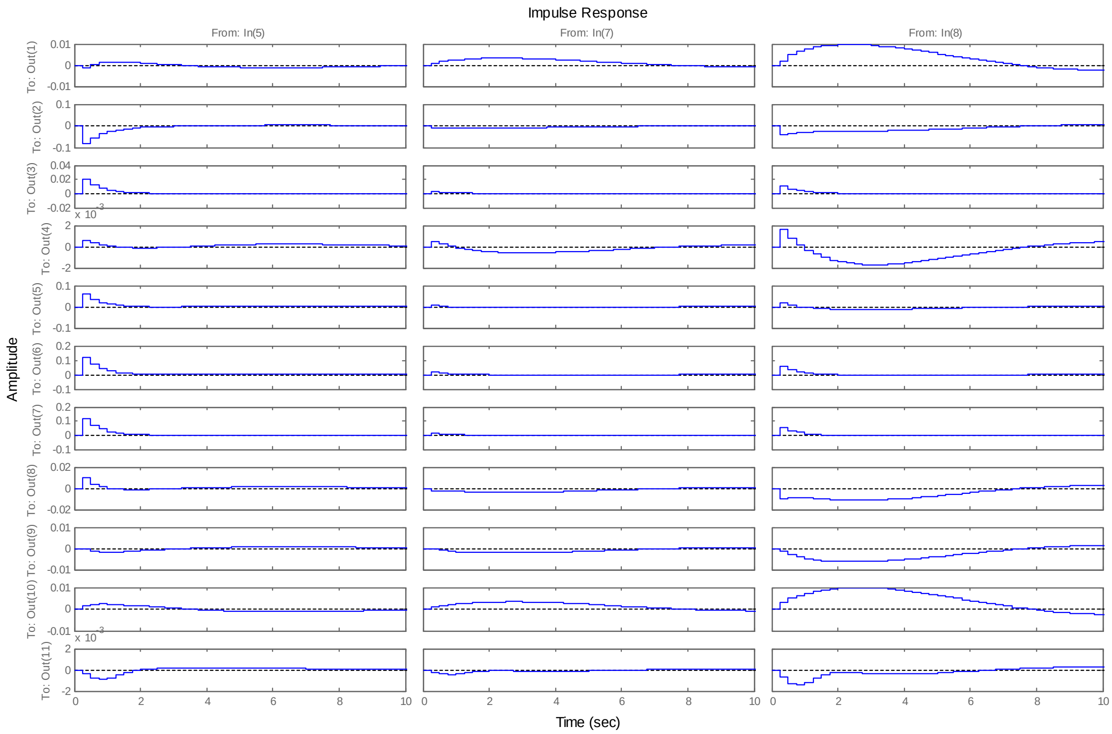
Responses to π^{**N}, p^{**X}, and p^{**A} :

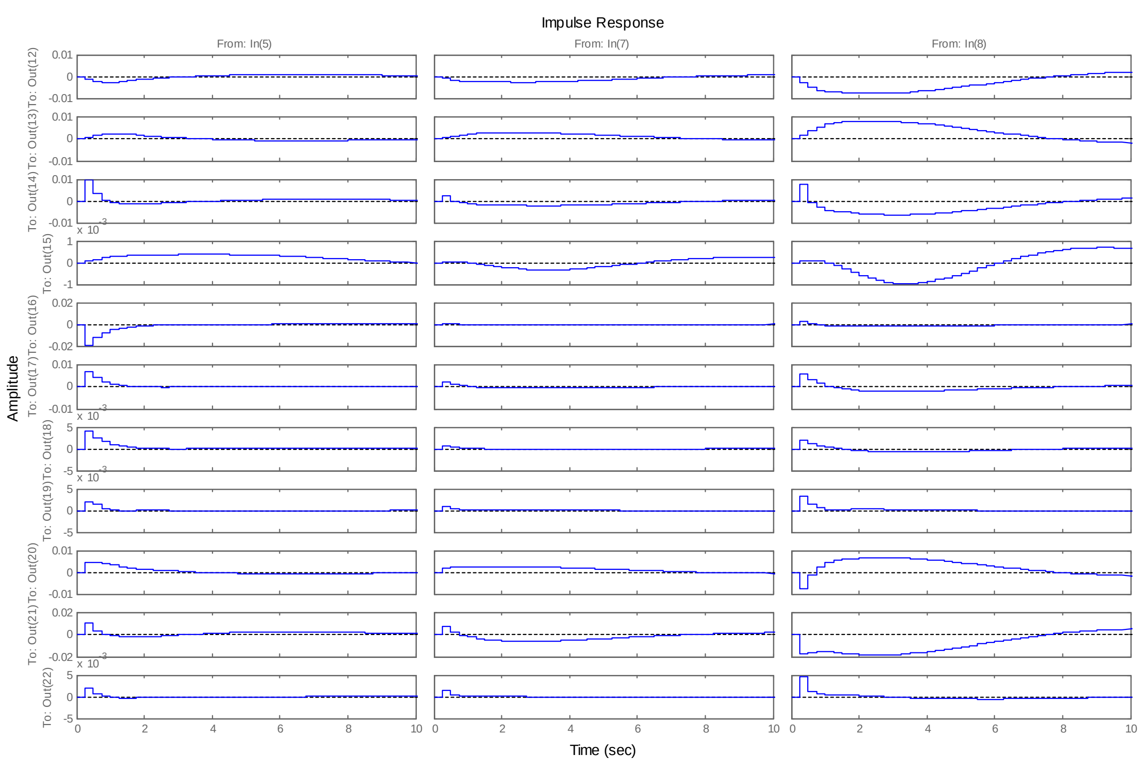


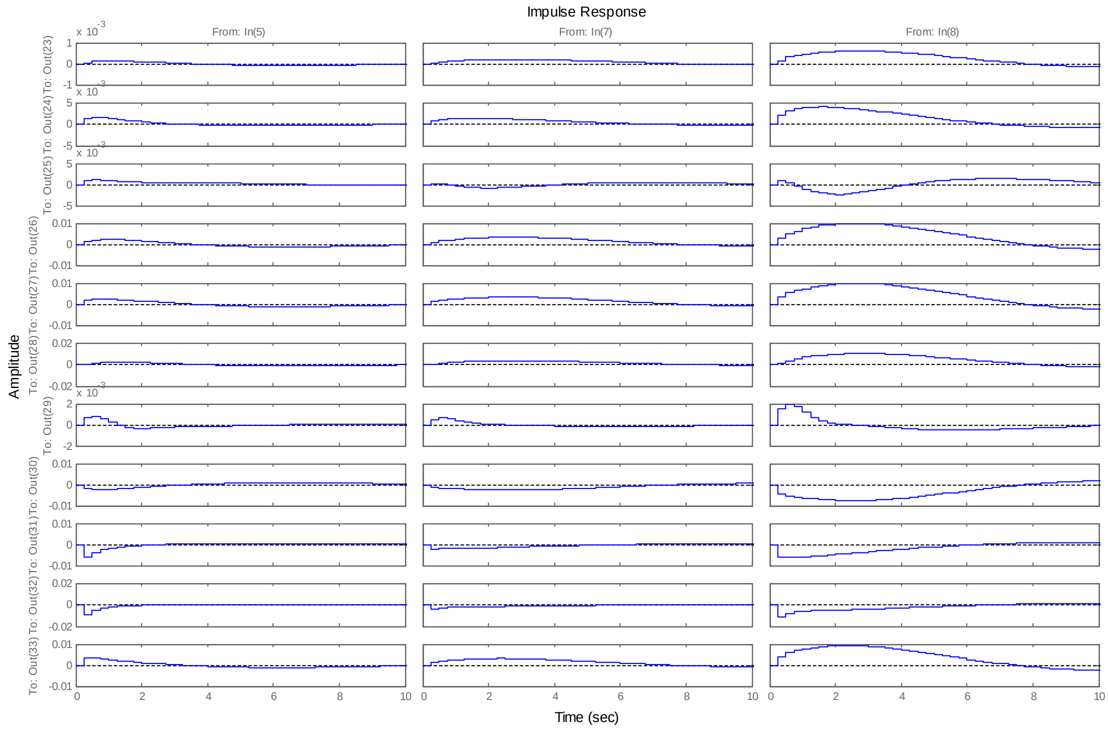

Impulse Response
From: In(15)
From: In(16)
From: In(17)
To: Out(12)
To: Out(13)
To: Out(14)
To: Out(15)
To: Out(16)
To: Out(17)
To: Out(18)
To: Out(19)
To: Out(20)
To: Out(21)
To: Out(22)
Amplitude
Time (sec)


Impulse Response
From: In(15)
From: In(16)
From: In(17)
To: Out(23)
To: Out(24)
To: Out(25)
To: Out(26)
To: Out(27)
To: Out(28)
To: Out(29)
To: Out(30)
To: Out(31)
To: Out(32)
To: Out(33)
Amplitude
Time (sec)

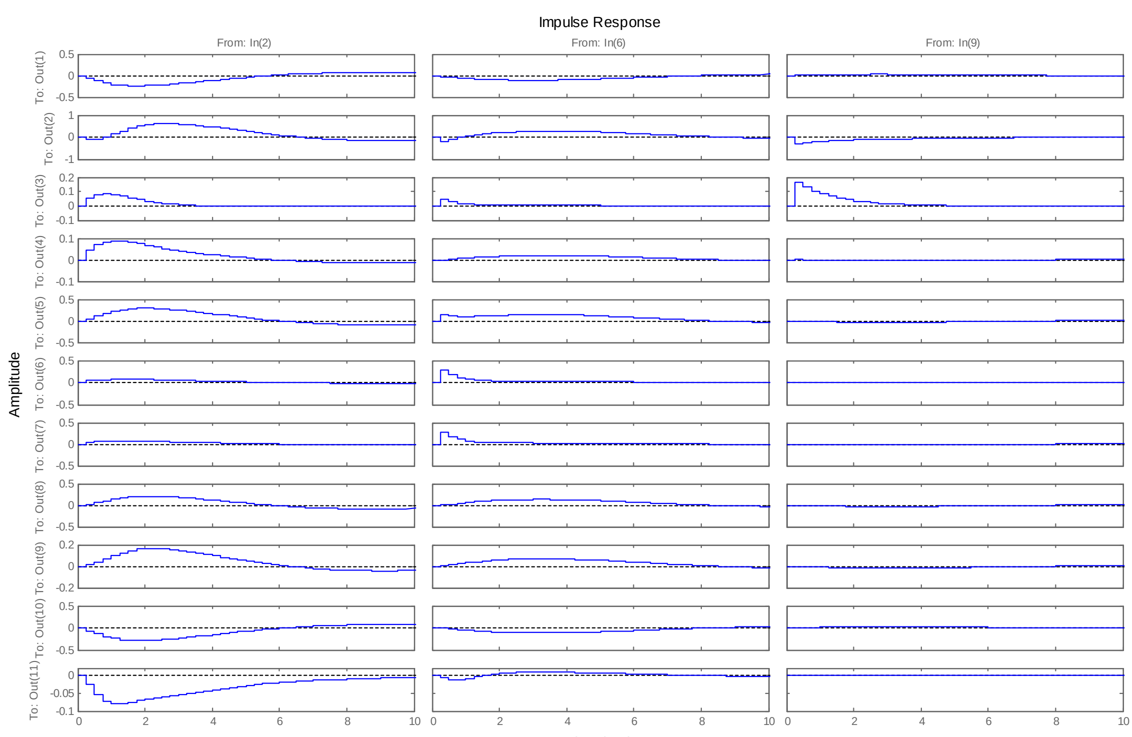
Responses to ς^K, ς^A, and ς^N :

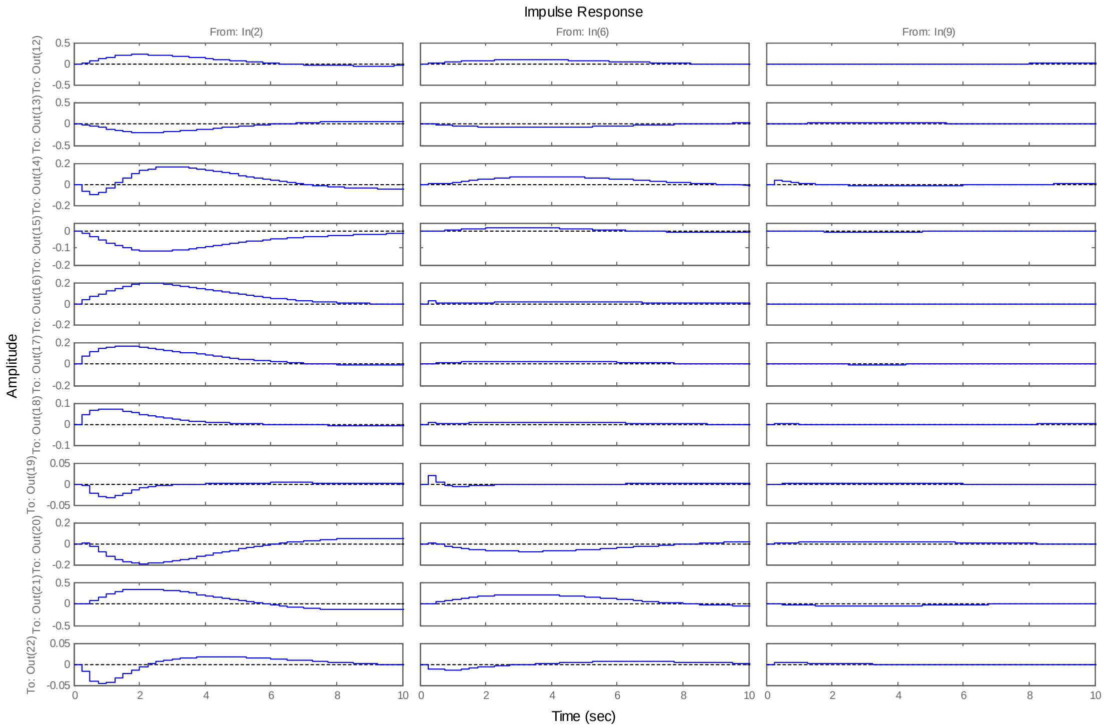


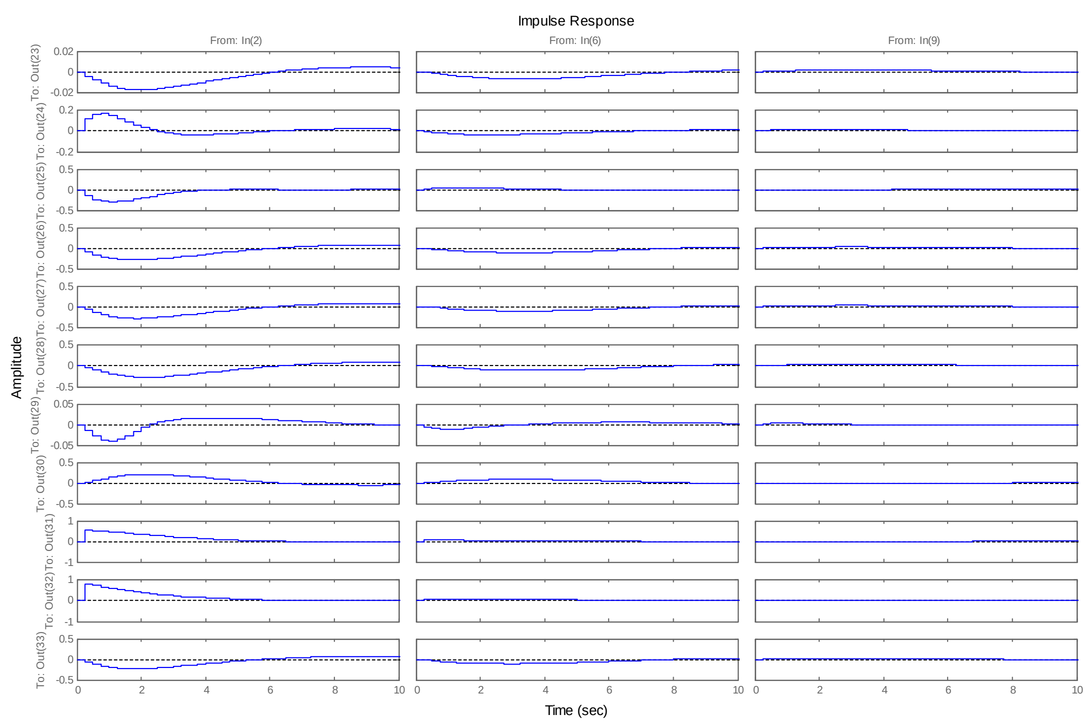


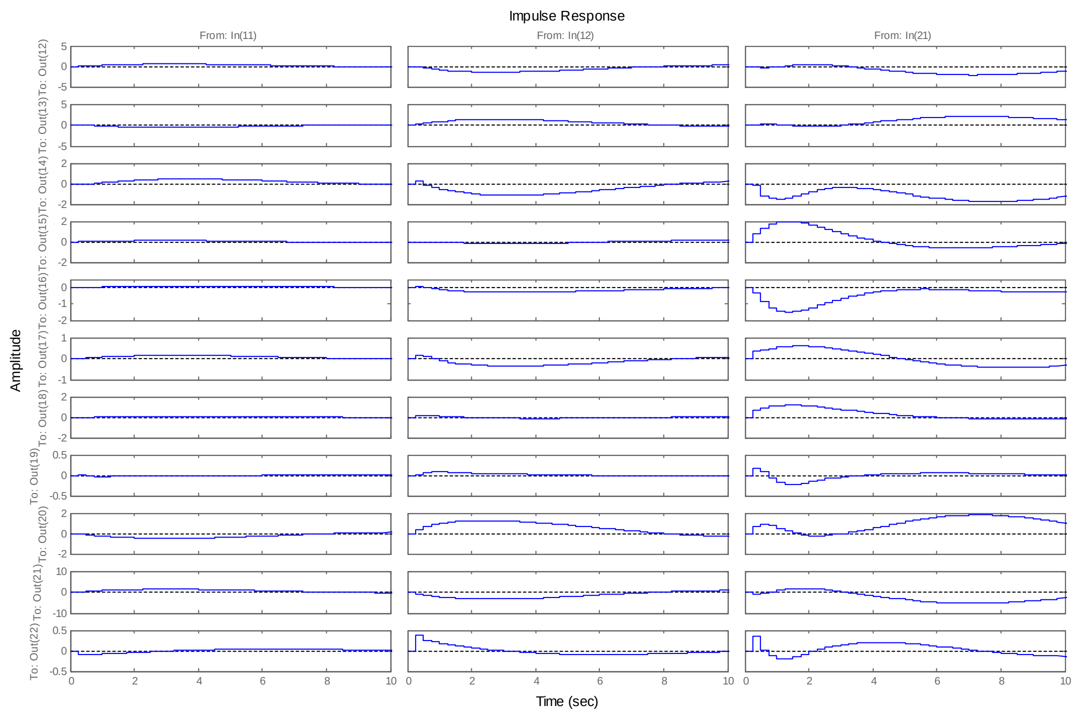


Responses to z^C, ς^W, and γ^B :

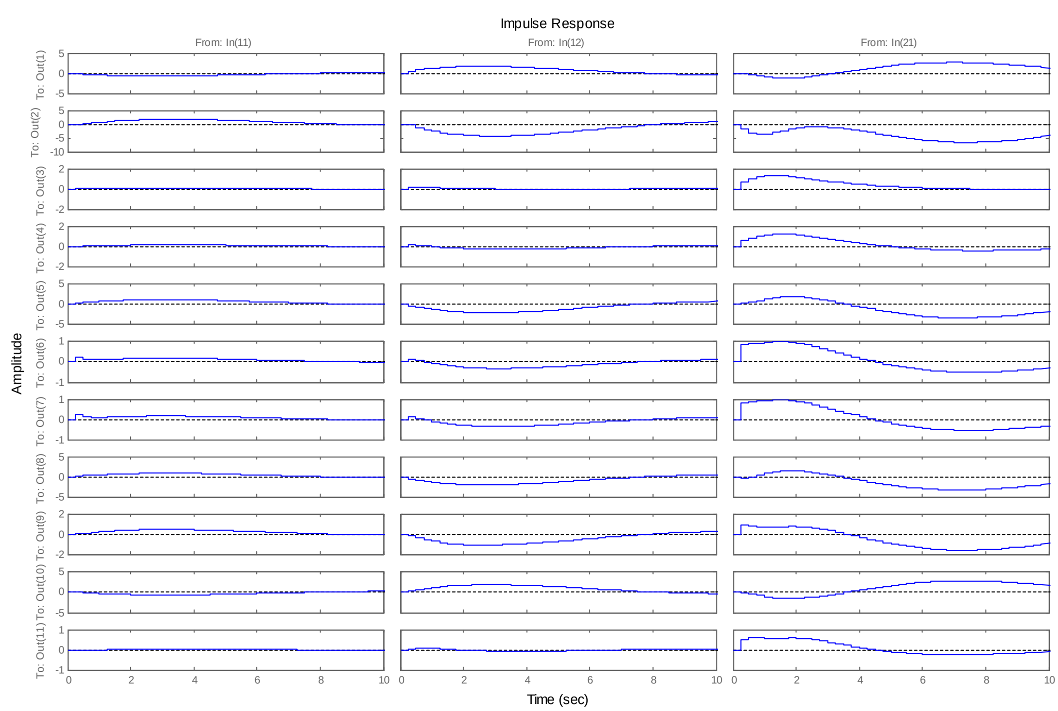


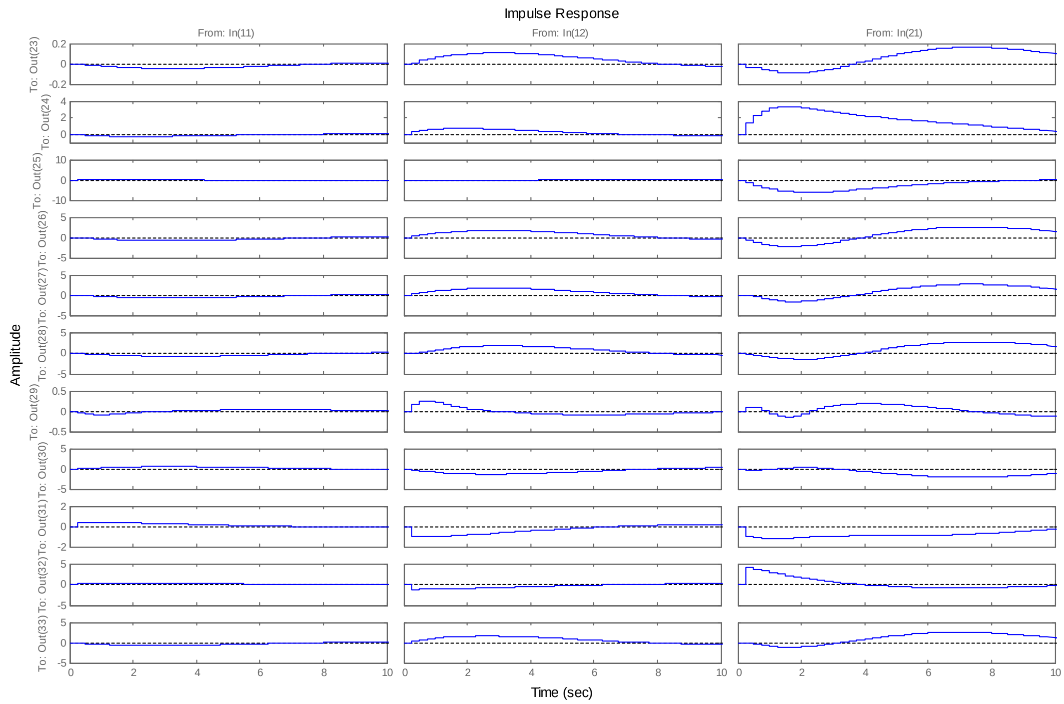

Impulse Response
From: In(2)
From: In(6)
From: In(9)
To: Out(12)
To: Out(13)
To: Out(14)
To: Out(15)
To: Out(16)
To: Out(17)
To: Out(18)
To: Out(19)
To: Out(20)
To: Out(21)
To: Out(22)
Amplitude
Time (sec)


Impulse Response
From: In(2)
From: In(6)
From: In(9)
To: Out(23)
To: Out(24)
To: Out(25)
To: Out(26)
To: Out(27)
To: Out(28)
To: Out(29)
To: Out(30)
To: Out(31)
To: Out(32)
To: Out(33)
Amplitude
Time (sec)



Responses to ℓ^G, g, and z^0 :





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