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Alternative Monetary Regimes in a DSGE Model
of a Small Open Economy with Two Sectors and
Sticky Prices and Wages

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Alternative Monetary Regimes in a DSGE Model of a Small Open economy with sticky Prices and Wages¹

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JEL Classification: E52, F41, F31

Abstract: This paper develops a dynamic stochastic general equilibrium (DSGE) model for a small open economy (SOE) that can be calibrated to simulate the macro dynamics of a semi-industrialized developing country like Argentina. We consider a multilateral non-commodity trade environment, with the U.S.A. and Europe as trade partners, and assume that the Law of One Price does not hold for the goods that the U.S.A. and Europe trade between them. We show that this makes the U.S.A.'s multilateral real exchange rate (MRER) a key fundamental for the SOE's MRER, in addition to its terms of trade. The SOE produces and consumes exportable and non tradable goods using labor (and in the case of exportables, imports). There is a representative, perfectly competitive firm producing exportables and operating under perfectly flexible export and import prices. Monopolistic competition with price (wage) stickiness prevails for non-tradable firms (households). These set prices (wages) subject to a price/wage adjustment cost function. There coexist both forward and backward looking firms. The latter use a "rule of thumb" to change prices that gradually corrects their price to that of optimizing firms. Alternative monetary or foreign exchange policy rules, including a fixed exchange rate, inflation targeting under a pure float and inflation targeting under a managed float, complete the dynamic systems. The non-stochastic steady state is analyzed in detail for the alternative models and the log-linearized systems are obtained.

1. Introduction

This paper is motivated by the desire to have a calibrated model of the Argentinian economy that could be used for policy simulations and analysis and also as a complement to the small structural model currently used by the Research Department of the Central Bank to make consistent projections of the key macroeconomic variables. The paper develops a dynamic stochastic general equilibrium (DSGE) model for a small open economy (SOE) that can be calibrated to simulate the macro dynamics of a semi-industrialized developing country like Argentina. It is built so as to accommodate various alternative monetary/exchange rate policies.

The model assumes rational expectations and optimizing behavior by a subset of the agents involved, who coexist with agents who make decisions based on "rules of thumb". The SOE produces and consumes exportable and non-tradable goods using labor (and in the case of exportables, imports). There is a representative, perfectly competitive and optimizing firm producing exportable goods. For simplicity, we assume that the Law of One Price prevails for the exports and imports of the SOE, with full and instantaneous pass-through of nominal depreciations to peso

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prices.² We leave out the sector that produces import-competing goods. Non-tradable sector firms are monopolistic competitors operating under price stickiness. A subset of these firms set optimal prices subject to a simple price adjustment cost function (as in Rotemberg (1982) and Sbordone (1998)). To account for inflation inertia (see Fuhrer and Moore (1995), Roberts (1997), Galí and Gertler (1999)), we introduce a segment of "rule of thumb" firms that both 1) index their prices to the previous period's non tradable inflation, and 2) correct a fraction of the discrepancy between their own, backward looking non-tradable price level and the (forward looking) non-tradable price level of optimizing firms. This generates a "hybrid Phillips curve" equation for non-tradable inflation that has some of the usual properties: in particular, the sum of the coefficients for lagged and expected inflation is in the interval between the intertemporal discount factor (β) and one. In this equation the rate of inflation also depends on the gap between expected and (non-stochastic) steady state marginal cost (which is standard) and on the lagged relative price between forward and backward looking non-tradable goods (which is not standard). When this relative price is eliminated (using its law of motion), the resulting "hybrid Phillips curve" has coefficients that are less restricted than in the usual versions. These coefficients critically depend on the exogenous rate of correction of relative price discrepancies of "rule of thumb" firms (α).

We consider a multilateral non-commodity trade environment, with the U.S.A. and Europe as the SOE's trade partners and make the critical assumption that the Law of One Price does not hold for the goods that the U.S.A. and Europe trade (within the model's time horizon). We show that, due to non tradable price stickiness, this makes the U.S.A.'s multilateral real exchange rate (MRER) a key fundamental for the SOE's MRER, along with (and separately from) its terms of trade. This has been shown to be empirically very significant in the case of Argentina (see Garegnani and Escudé (2005)).

Households consume both non-tradable and exportable goods under habit and do not consume imported goods. In Argentina, as in most developing countries, imports basically consist of inputs to production, with little participation of consumption goods. Hence, the assumption that households do not consume imported goods seems an acceptable first approximation (see McCallum and Nelson (2000)). One could think of the exportable/non-tradable sectors as roughly representing the classification of output into goods (which are in general exportable) and services (which are in general non tradable). However, there is usually some degree of market power and price stickiness in some of the goods producing sectors (particularly the manufacturing sector). Because our exportable sector is perfectly competitive and has perfectly flexible prices we do not associate the non-tradable/exportable partition with the goods/services partition. On the other hand, it is not very realistic to assume that the exportable sector is perfectly competitive, as there is usually some degree of price discrimination and market segmentation for manufactured exportable goods. Nevertheless, we have chosen to make the perfect competition assumption as a first approximation that makes the model more tractable, leaving both monopolistic competition and sticky price setting for the non-tradable sector.

Households are monopolistic competitors in labor supply. They set their own

²Non-commodity exports under local currency pricing and sticky price setting are considered in a sequel to this paper (see Escudé (2006)).

wage under a wage adjustment cost function and this generates a forward looking dynamic wage inflation equation. They can hold money and domestic currency (Central Bank) bonds, and do not incur in debt. To simplify, we assume that foreign currency Government bonds and Central Bank international reserves are dollar denominated. The Government has a fiscal policy characterized by exogenous paths for taxes and expenditures, while it finances any deficit by issuing dollar denominated bonds abroad. Foreign investors demand a risk premium for purchasing Government bonds. This premium is assumed to have an exogenous component as well as an endogenous component that varies positively with the public sector's net foreign currency debt. Money demand is introduced through a stylized transactions technology where holding money saves on transaction costs in terms of the exportable or non-tradable good that is transacted. Arbitrageurs make the uncovered interest parity condition hold between domestic currency (Central Bank) bonds and dollar denominated (Government) bonds.

Instead of adopting the widely used Calvo (1983)-Rotemberg (1987)-Yun (1996) staggered pricing cum indexation framework, we use adjustment cost functions similar to those in Rotemberg (1982, 1994) and Sbordone (1998). The adjustment costs presumably reflect the use of resources in the process of optimal decision making, such as information gathering and analysis, evaluation of customers' possible reactions, etc. In contrast to most papers that use such functions, our non-linear model fully accounts for the consumption of real resources during the decision making processes. The use of such resources is eliminated outright in the Calvo (1983) framework by the exogenous stochastic process that determines which firms can optimize in any given period of time. In the end there is no substantial gain in realism, however, since in our setting these costs are of second order and hence disappear upon log-linearization of the model. We do believe that there is a gain in realism in the introduction of firm heterogeneity through the existence of "rule of thumb" firms that choose not to undertake costly optimization decision processes. Firm heterogeneity is more essential in our framework than in the Calvo-Rotemberg-Yun approach because in the latter all firms will eventually set optimal prices, whereas in our framework "rule of thumb" firms never optimize, making it necessary to keep track of the relative price between the two kind of firms. This leads to a "hybrid" Phillips equation for non-tradables that has less restricted coefficients for expected and lagged inflation than other formulations. The Phillips equations for non-tradable (core) inflation and wage inflation reflect a gradual adjustment of non-tradable goods inflation and wage inflation towards their long-run levels.

We close the model with four alternative monetary/exchange rate policies: 1) a fixed exchange rate with a single currency (the U.S. dollar), 2) a fixed exchange rate with a trade weighted basket of currencies, 3) inflation targeting under a pure float, and 4) inflation targeting under a managed float. The Central Bank is assumed to have a policy of handing over any "quasi-fiscal" surplus or deficit to the Government and thus keeping a balance sheet that in each period fully backs monetary and domestic currency bond liabilities with international reserves. This assumption plays a key role in generating a clearly defined supply of Central Bank bonds and allows for the possibility of inducing private sector portfolio shifts through the simultaneous use of money market and foreign exchange market interventions

in the Inflation Targeting with Managed Float regime. In the latter regime, the Central Bank simultaneously uses an interest rate feedback rule and a feedback rule for the use of international reserves in foreign exchange market interventions. The latter feedback rule reflects a policy of "leaning against the wind" by purchasing foreign exchange when the currency tends to appreciate (see McCallum (1994)).

The stochastic processes driving the exogenous variables interact with agents' decisions to determine the system's dynamics. We introduce a fairly large amount of exogenous variables in the theoretical model, some of which can easily be removed in specific simulations: the U.S.A.'s MRER, the terms of trade, the exogenous component of the risk premium, government expenditures, the international interest rate, sectorial productivity shocks, a labor supply shock, and an openness shock. We analyze the non-stochastic steady states of the alternative systems and subsequently log-linearize the model's equations and put the system in a form suitable for the use of known solution and estimation methods. The productivity process could be used to incorporate exogenous per capita growth. However, we have chosen not to do so since we believe that the model's long run should be interpreted as a medium run. In this time frame the structure of trade is taken as fixed and the Law of One Price is assumed not to hold between the traded goods of the U.S.A. and Europe. This helps to capture the real effects of the international strengthening of the dollar when there is a fixed exchange rate regime that pegs to the dollar.³

The resulting dynamic model has a non-stochastic steady state (i.e. a steady state where the stochastic exogenous driving variables are at their unconditionally expected value) in which there is full wage and price flexibility, i.e. there are no price and wage adjustment costs. This steady state is similar to the static Blanchard and Kiyotaki (1987) model, except for the fact that it represents a two-sector small open economy. Our dynamic system has similarities to those in Erceg, Henderson and Levin (2000), Christiano, Eichenbaum and Evans (2001), Smets and Wouters (2002) and Benigno and Woodford (2005), insofar as there is both price and wage stickiness, but all these papers are for closed economies. There are also similarities with many papers in the now vast literature on open economy monetary policy. In particular, Galí and Monacelli (2003), have a one sector small open economy in a world of small open economies. They do not have monopolistic competition in the household sector, and the rest of the world is explicitly modeled. We abstain from modeling the rest of the world. Our framework also has many similarities with Devereux and Lane (2003), who have a two sector SOE that produces exportables and non tradables, with perfect competition in the first and monopolistic competition in the second with a Calvo-Rotemberg-Yun approach to price stickiness. Their households, however are wage takers and consume non tradables and imports (instead of exportables) while all exportable production is exported. Their framework is more complicated than ours in some respects,

³That these real effects are substantial has been captured in a recent empirical paper (Garegnani and Escudé (2004)) where it is shown that Argentina's terms of trade index and the U.S.A's MRER (as measured by the Federal Reserve's Real Broad Dollar Index) can explain most of the short and long run dynamics of Argentina's MRER in a single equation Equilibrium Correction Mechanism framework. That paper shows that the ECM coefficient is statistically significant only outside the time periods in which the peso was pegged to the dollar.

since they have investment and firms that produce non-finished capital goods, as well as entrepreneurs that supply sector specific entrepreneurial labor and produce the final capital goods. Also, although they assume the Law of One Price for export goods as we do, they allow for a gradual pass-through for import goods, since an important concern for them is the relative performance of two inflation targeting rules (which target CPI and non tradable inflation, respectively) and a fixed exchange rate regime, under different speeds for pass-through. Another difference is that we are interested in further formalizing the "strong dollar shock" that we used in Escudé (2004a) and Escudé (2004b) and measured in Garegnani and Escudé (2004), for which we need an explicit multilateral trade and finance framework.

We recur to Occam's razor and omit the virtual "benchmark flexprice economy" that combines price and wage flexibility with current values of the forcing processes to generate a theoretical "output gap" (based on a theoretical "natural" output level) and a theoretical "natural interest rate" (see Rotemberg and Woodford (1999) and Woodford (2003)). While these concepts have an intuitive appeal when the model is highly stylized, they become less attractive as soon as one starts introducing realistic complications. In particular, the coexistence of price stickiness with wage stickiness renders these concepts less clear cut than when there is only price stickiness (leading Benigno and Woodford (2005) to introduce a "natural wage"). Matters get even more complicated when we introduce "rule of thumb" agents. Furthermore, it is difficult to find a clearly measurable variable that could proxy the "natural" output level. The usual procedures for deriving a level of "potential" output seem to better proxy the (growth adjusted) non-stochastic steady state, which reflects the average values of the forcing variables. Indeed, such a state would be observable if the exogenous forcing variables all happened to coincide with their mean values for a sufficiently long period of time. Hence, smoothing techniques as those used to obtain practical estimates of "potential output" (either direct, or indirect as in the "production function approach") may be seen as a way of empirically approximating such states. However, the "natural" rates which would prevail in a hypothetical world devoid of nominal price rigidities but subject to the usual stochastic shocks are extremely difficult to measure empirically (see Amato (2005)) even if not impossible (see Neiss and Nelson (2002)). The important fact that the "natural" rate of output naturally appears in second order approximations to household utility (Rotemberg and Woodford (1998),(1999)) does not seem to require keeping inventory of "natural" rates for all the endogenous variables in the model. For all of these reasons we have chosen to leave the "natural" rates (and respective "gaps") out of the picture. Consequently, the log-linear approximation to the non-linear model explicitly includes all the shocks that affect the system.

The rest of the paper is as follows: Section 2 presents the basic relative price definitions and the decision processes of the different kinds of firms. Section 3 presents the decision process for households. Section 4 has the market clearing conditions. Section 5 describes the public sector and the balance of payments. Section 6 puts together the complete (monetary regime dependent) non-linear systems and shows their non-stochastic steady states. Section 7 presents the corresponding log-linearized systems. Section 8 elaborates on the inflation targeting with managed float regime. Section 9 shows how some typical macroeconomic equations (like the

IS and LM) can be derived from the model. Finally, Section 10 concludes. An Appendix works out in detail the log-linearization of non-tradable firms' first order conditions that results in the Phillips curve.

2. Firm decisions

2.1 The MRER and other relative prices

There are two categories of goods produced by the SOE: exportable and non-tradable goods (or services). Since these are also the goods consumed by households, it is natural to define the multilateral real exchange rate (MRER) as the relative price between exportable (X) and non-tradable (N) goods. We assume that absolute purchasing power parity holds, with full and immediate pass-through. Hence, the MRER (e_t) is defined as

$$e_t \equiv \frac{\phi_t S_t^m}{P_{N,t}},$$

where S_t^m is the multilateral nominal exchange rate (pesos per a geometrically trade weighted basket of currencies), ϕ_t is the geometrically trade weighted basket of export price indexes, and $P_{N,t}$ is the peso price of non-tradables.

In many less developed countries the nominal exchange rate is sometimes fixed or pegged to a single hard currency in which a large part of either commercial or financial transactions are carried out.⁴ Assuming that we have a small economy that is a price taker in the international markets and that a significant part of its trade is in manufactured goods, where the Law of One Price need not hold between the tradable goods of the SOE's trading partners, changes in the reference country's MRER are a potential source of shock to the SOE.⁵

Among other monetary/exchange rate regimes, we consider the unilateral fixing of the exchange rate to a single currency, that we take as the U.S. dollar. For simplicity, we reduce the SOE's trade partners to the U.S.A. and Europe (which thus represents all trade partners except the U.S.A.), concentrate on non-commodity trade, and assume that a significant fraction of trade (α_{EU}) is done with the Euro area and the rest ($\alpha_{US} = 1 - \alpha_{EU}$) with the U.S.A., and that these coefficients hold both for exports and imports. Furthermore, they are constant under the assumption that the time it takes to significantly change the structure of trade is longer than the model's long run.⁶ The MRER can be defined as a geometrically weighted average of bilateral real exchange rates (first equality), or equivalently, as a ratio between the multilateral nominal exchange rate and the non-tradables price index

⁴Less often, the nominal exchange rate is pegged to a basket of currencies.

⁵This issue was particularly relevant in the case of Argentina's fixed exchange rate with the U.S. dollar during its "Convertibility" regime, which played a major role in leading to the worst crisis in 100 years when the dollar persistently appreciated in real terms between 1995 and 2001, wreaking havoc in Argentina's manufacturing sector and generating massive unemployment. Indeed, the two times Argentina pegged to the U.S. dollar during the last 30 years (the "tablita" period in the late 70s and the "Convertibility" period) ended in very costly triple crises after a lengthy period of dollar strengthening.

⁶This seems reasonable in a model where there is no investment nor growth.

(second equality):

$$e_t = \left(\frac{S_t P_t^{US}}{P_{N,t}} \right)^{\alpha_{US}} \left(\frac{(S_t/\rho_t^*) P_t^{EU}}{P_{N,t}} \right)^{\alpha_{EU}} = \frac{\phi_t S_t / \rho_t}{P_{N,t}} \quad (1)$$

where P_t^{US} and P_t^{EU} are the price indexes of the U.S.A. and Europe, S_t is the peso/dollar nominal exchange rate, ρ_t^* is the exogenous euro/dollar nominal exchange rate,

$$\rho_t \equiv (\rho_t^*)^{\alpha_{EU}} \equiv (1)^{\alpha_{US}} (\rho_t^*)^{\alpha_{EU}} \quad (2)$$

is the exogenous trade weighted basket of foreign currencies per dollar nominal exchange rate ("dollar strength"),

$$S_t / \rho_t = (S_t)^{\alpha_{US}} (S_t / \rho_t^*)^{\alpha_{EU}}$$

is the SOE's multilateral nominal exchange rate (that we represented as S_t^m previously), and

$$\phi_t \equiv (P_t^{US})^{\alpha_{US}} (P_t^{EU})^{\alpha_{EU}}$$

is the export price index as well as the terms of trade because we assume that there is no inflation in import prices and that the multilateral import price index is normalized to one. Because we assume that firms produce, and households consume, exportable and non-tradable goods, e_t is the relevant relative price for output decisions as well as consumption decisions.

The consumption sub-utility function will have a Cobb-Douglas specification for the consumption of exportable and non-tradable goods. Hence, the (dual) Consumer Price Index is a Cobb-Douglas index of these goods' prices:

$$P_t = (\phi_t S_t / \rho_t)^\theta (P_{N,t})^{1-\theta}, \quad (3)$$

where $0 < \theta < 1$. Let $w_t \equiv W_t / P_{N,t}$ be the product wage in the non-tradable sector, where W_t is the index for nominal wages. Then the product wage in the exportable sector is

$$\frac{W_t}{\phi_t S_t / \rho_t} = \frac{W_t / P_{N,t}}{(\phi_t S_t / \rho_t) / P_{N,t}} = \frac{w_t}{e_t}. \quad (4)$$

We will often find convenient to use the domestic purchasing power of the dollar: $s_t \equiv S_t / P_t$. The following relation follows from (1) and (3):

$$s_t = \frac{\rho_t}{\phi_t} e_t^{1-\theta}. \quad (5)$$

Also, note that the real wage in terms of the consumption basket is:

$$w_t^\circ \equiv \frac{W_t}{P_t} = \frac{w_t}{e_t^\theta}. \quad (6)$$

Furthermore, the fact that the usual definition of the MRER uses the CPI in the denominator is of little consequence. If we define $e_t^\circ = (\phi_t S_t / \rho_t) / P_t$ we obtain $e_t^\circ = e_t^{1-\theta}$. Hence, we could work with this alternative concept of the MRER in all that follows by merely replacing e_t by $(e_t^\circ)^{-1/(1-\theta)}$.

2.2 Technology and the labor market

The two production sectors in the model produce exportable (X) and non-tradable (N) goods, respectively. We assume that capital is fixed in each sector and does not depreciate and that labor is perfectly mobile between sectors but immobile internationally. There is a representative firm in the export sector and a continuum of monopolistically competitive firms in the non-tradable sector, each of which is characterized by the non-tradable variety $i \in [0,1]$ it produces. Output in each sector is related to the labor input and technology shocks by the respective production functions:

$$y_{X,t} = z_t^X F_X(L_{X,t}), \quad y_{N,i,t} = z_t^N F_N(L_{N,i,t}). \quad (7)$$

These production functions have positive and diminishing marginal labor productivity, where z_t^F ($F = X, N$) is an exogenous productivity shock which is common to all firms in sector F , L_X and $L_{N,i}$ are aggregates of the complete range of labor types $j \in [0,1]$, as we will see in the next section, and in particular $L_{N,i}$ is the amount of the labor aggregate used by non-tradable sector firm i . We assume that the production of one unit of exportable good also requires ϵ_t units of imported goods, where ϵ_t is a positive and possibly time-varying coefficient. Hence, the relation between import and labor inputs in the exportable sector is given by $I_{X,t} = \epsilon_t z_t^X F_X(L_{X,t})$, where $I_{X,t}$ is the total import requirement.⁷ Note that (under our assumption that $P_M^* = 1$) the nominal value of net exports is:

$$(S_t/\rho_t)(\phi_t - \epsilon_t)y_{X,t}. \quad (8)$$

We assume that there is a single labor market where all firms (whether in the non-tradable or export sector) hire the same CES aggregate of all types of labor and face the same wages. As in Erceg et al (2000), we assume that there is a competitive ‘‘employment agency’’ (or ‘‘representative labor aggregator’’) that combines households’ labor types in the same proportion that firms would choose. Define the aggregate of labor types as:

$$L_t = \left(\int_0^1 (L_t^h)^{\frac{\psi-1}{\psi}} dh \right)^{\frac{\psi}{\psi-1}} \quad (\psi > 1). \quad (9)$$

We will refer to L_t as ‘labor’. The employment agency’s demand for each labor type h is equal to the sum of all firms’ demands. It minimizes the cost of employing a given level of L_t . Hence, it minimizes

$$\int_0^1 W_t^h L_t^h dh$$

subject to (9) for a given value of L , where W_t^h is the wage rate set by the monopolistic supplier of labor type h . This gives the agency’s demand (and the aggregate demand of all firms) for labor type h as:

$$L_t^h = L_t \left(\frac{W_t^h}{W_t} \right)^{-\psi} \quad (10)$$

⁷The production function in the exportable sector is hence $y_X = G(L_X, I_X) \equiv \min(z_t^X F_X(L_X), I_X/\epsilon)$. There will be no restrictions on imports in this paper, so we simply keep the import requirement separate from the ‘‘partial’’ production function $z_t^X F_X(L_X)$.

where W_t is the aggregate wage index, defined as:

$$W_t = \left(\int_0^1 (W_t^h)^{1-\psi} dh \right)^{\frac{1}{1-\psi}}, \quad (11)$$

and ψ is the elasticity of substitution between differentiated labor services. The higher ψ is, the lower is the monopolistic power of households, because the varieties of labor are closer substitutes. Total labor cost is given by

$$\int_0^1 W_t^h L_t^h dh = W_t L_t.$$

2.3 The exportable goods sector

The exportable goods producing sector is assumed to be perfectly competitive. It has a representative firm that chooses labor and imports each period so that its marginal productivity is equal to the product wage (4). Using (8), nominal profit in the export sector is:

$$(S_t/\rho_t)(\phi_t - \epsilon_t)z_t^X F_X(L_{X,t}) - W_t L_{X,t}.$$

Hence, the first order condition for profit maximization is:

$$F'_X(L_{X,t}) = \frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t}. \quad (12)$$

We assume that ϵ_t is strictly less than ϕ_t at all times. Otherwise, the exportable sector would disappear. Given w_t/e_t , labor demand by the exportable sector is increasing in z_t^X and ϕ_t/ϵ_t .

2.4 The non-tradable goods sector

2.4.1 The forward looking Phillips equation for core inflation

Each firm in the non-tradable sector is constrained in its price setting activity by the fact that changing price is costly. For simplicity, we assume that this price changing activity requires the non utility generating consumption of the non-tradable whose price is to be adjusted. As in Rotemberg (1994) and Sbordone (1998), let $x(\log \pi_{N,i,t})$ represent the cost per unit sale of changing $P_{N,i,t-1}$ at the rate $\pi_{N,i,t} \equiv P_{N,i,t}/P_{N,i,t-1}$. We assume that this adjustment cost function has the following properties:

$$x(\log \bar{\pi}_N) = x'(\log \bar{\pi}_N) = 0, \quad x''(\log \bar{\pi}_N) = a_F > 0. \quad (13)$$

where $\bar{\pi}_N$ denotes the non-stochastic steady state gross level of inflation, and is always determined by the Central Bank's monetary/exchange rate regime (and targets), as we shall see.⁸ Each firm in the non-tradable sector is also constrained

⁸A quadratic adjustment cost function, as used in Rotemberg (1994) is a particular case for $x(\cdot)$. Note that in general we do not need symmetry between the costs of upward and downward adjustments, such as the quadratic function implies. According to Rotemberg (1994), "the model with quadratic costs of changing prices is equivalent, as far as the aggregates are concerned, to a model such as Calvo (1983) where individual firms have a constant hazard of adjusting their price." Also, note that while Rotemberg (1994) assumes that the costs of changing prices "do not reduce the output available for consumption" we explicitly model this reduction in the non-linear model. However, these costs disappear in the log-linearization, since they are of second order. Furthermore, Sbordone (1998) multiplies the convex cost function times the aggregate value of output, instead of the individual value of output as we do in (17). With this exception, however, our derivation of the forward looking Phillips equation is very similar to Sbordone's.

by its technology and by the demand function it faces for its distinct variety i , which will be derived in the next section:

$$y_{N,i,t} = y_{N,t} \left(\frac{P_{N,i,t}}{P_{N,t}} \right)^{-\nu}. \quad (14)$$

The aggregate non-tradable price level is given by the Dixit-Stiglitz price index:

$$P_{N,t} = \left\{ \int_0^1 (P_{N,i,t})^{1-\nu} di \right\}^{1/(1-\nu)}, \quad (15)$$

where ν is the elasticity of substitution between differentiated non-tradable goods. The higher ν is, the lower is the market power of firms because the varieties are closer substitutes. Firm i chooses $P_{N,i,t}$ to maximize the expected present value of present and future profits:

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Pi_{i,t+j}^N \quad (16)$$

where

$$\Pi_{i,t}^N = P_{N,i,t} y_{N,i,t} \left\{ 1 - x \left(\log \left(\frac{P_{N,i,t}}{P_{N,i,t-1}} \right) \right) \right\} - W_t L_{N,i,t}, \quad (17)$$

$$\Lambda_{t,t+j} \equiv \prod_{k=1}^j \frac{1}{1 + i_{t+k}}, \quad (j > 0), \quad \Lambda_{t,t} \equiv 1, \quad (18)$$

and $y_{N,i,t}$ satisfies the technological and demand constraints ((7) and (14)). The result of this maximization (which is detailed in the Appendix) is:

$$G_t^P = \mu_F \frac{w_t}{F_N'(F_N^{-1}(y_{N,t}/z_t^N))}, \quad \left(\mu_F = \frac{\nu}{\nu - 1} \right) \quad (19)$$

where the (inverse) markup gap G_t^P is defined by

$$G_t^P \equiv 1 - x(\log \pi_{N,t}) + \frac{1}{\nu - 1} \left\{ x'(\log \pi_{N,t}) - E_t \left[\Lambda_{t,t+1} \frac{y_{N,t+1}}{y_{N,t}} \pi_{N,t+1} x'(\log \pi_{N,t+1}) \right] \right\}. \quad (20)$$

Because in this subsection all non-tradable firms face the same problem and hence set the same price, we have dropped the subscript i in (19) and (20). Note that (due to (13)) in the steady state the markup gap collapses to unity, which implies that the non-tradable price index is a constant markup μ_F over marginal cost:

$$\frac{1}{\bar{w}} = \mu_F \frac{1}{F_N'(F_N^{-1}(\bar{y}_N/\bar{z}^N))}. \quad (21)$$

Off the steady state we have a variable markup given by μ_F/G_t^P . Whenever G_t^P is greater (smaller) than one, the non-tradables markup is below (above) μ_F . Log-linearizing (19) and (20) (see the Appendix) yields a forward looking core (non-tradables) inflation "Phillips curve" equation:

$$\hat{\pi}_{N,t} = \beta E_t \hat{\pi}_{N,t+1} + \gamma_F \left\{ \hat{w}_t + a_y (\hat{y}_{N,t} - \hat{z}_t^N) \right\}, \quad (22)$$

$$\gamma_F \equiv \frac{\nu - 1}{a_F}, \quad a_F \equiv \frac{\bar{\varepsilon}_{F_N'}}{\bar{\varepsilon}_{F_N}} = \frac{-\bar{L}_N F_N''(\bar{L}_N) \bar{y}_N / F_N'(\bar{L}_N)}{\bar{L}_N F_N'(\bar{L}_N) \bar{y}_N / F_N(\bar{L}_N)}.$$

We generally use the notation $\hat{x}_t = \log(x_t/\bar{x})$ for the log deviation of x_t from its steady state value \bar{x} , and $\bar{\varepsilon}_f$ for the steady state value of the elasticity of the function $f(\cdot)$ with respect to its (only) argument. We have also used the fact that the steady state value of $\Lambda_{t,t+1}$ is equal to the intertemporal discount factor β . Equation (22) shows that the (log deviation of) non-tradable inflation varies positively with the expected (log deviation of) non-tradable inflation in $t+1$ and with the (log deviation of) real marginal cost in the non-tradable sector. The effect of changes in marginal cost is higher, the higher is the elasticity of demand ν and the lower is the convexity of the price adjustment cost function a_F .

2.4.2 The "hybrid" Phillips equation for core inflation

As is well known, the data indicate that there is not only price level inertia but also inflation rate inertia (see Fuhrer and Moore (1995), Roberts (1997) and Galí and Gertler (1999)). We now introduce firm heterogeneity in order to obtain a non-tradable Phillips equation that is both backward and forward looking (or "hybrid"). Let there be a fraction ς_F of non-tradable firms (those in the interval $[1, \varsigma_F]$) that are forward looking as above and whose price and inflation rate are $P_{N,t}^f$ and $\pi_{N,t}^f$. And assume that non-tradable firms in the interval $(\varsigma_F, 1]$ are completely backward looking monopolistic competitors that instead of making costly decisions for price changes have a "rule of thumb" for determining their price which is completely backward looking. These firms follow a simple indexation plus catch-up rule:

$$P_{N,t}^b = P_{N,t-1}^b [\pi_{N,t-1} + \alpha \bar{\pi}_N (p_{N,t-1} - 1)],$$

where $p_{N,t}$ is the relative price between optimizing and "rule of thumb" firms:

$$p_{N,t} = P_{N,t}^f / P_{N,t}^b. \quad (23)$$

Hence, "rule of thumb" firms have an inflation rate that 1) fully indexes to the general non tradable inflation rate, but also 2) corrects a fraction α (> 0) of the discrepancy between the current relative price with forward looking firms and the desired relative price (which is 1):

$$\pi_{N,t}^b = \pi_{N,t-1} + \alpha \bar{\pi}_N (p_{N,t-1} - 1). \quad (24)$$

The introduction of the "catching up" component is made in order to have a steady state where not only are the rates of inflation of backward and forward looking firms the same, but also their price levels. This treatment is thus somewhat different from the one in Galí and Gertler (1999). Using the fact (proved further below) that the steady state value of p_N is one, the log linear version of this equation is

$$\hat{\pi}_{N,t}^b = \hat{\pi}_{N,t-1} + \alpha \hat{p}_{N,t-1}. \quad (25)$$

Since there are only two types of firms (f and b) and in our framework the firms in either class are identical in all respects, (15) and (14) imply:

$$(P_{N,t})^{1-\nu} = \varsigma_F \left(P_{N,t}^f \right)^{1-\nu} + (1 - \varsigma_F) \left(P_{N,t}^b \right)^{1-\nu} \quad (26)$$

$$y_{N,t}^k = y_{N,t} \left(\frac{P_{N,t}^k}{P_{N,t}} \right)^{-\nu} \quad (k = f, b). \quad (27)$$

Let us define the relative prices and inflation rates

$$p_{N,t}^k = P_{N,t}^k / P_{N,t} \quad \pi_{N,t}^k = P_{N,t}^k / P_{N,t-1}^k \quad (k = f, b). \quad (28)$$

Then we may rewrite the previous equations as

$$1 = \varsigma_F \left(p_{N,t}^f \right)^{1-\nu} + (1 - \varsigma_F) \left(p_{N,t}^b \right)^{1-\nu} \quad (29)$$

$$y_{N,t}^k = y_{N,t} \left(p_{N,t}^k \right)^{-\nu}, \quad (k = f, b) \quad (30)$$

and obtain the following relations:

$$\pi_{N,t}^k = \left(p_{N,t}^k / p_{N,t-1}^k \right) \pi_{N,t} \quad (k = f, b)$$

$$\frac{p_{N,t}}{p_{N,t-1}} = \frac{\pi_{N,t}^f}{\pi_{N,t}^b}. \quad (31)$$

Log-linearizing (26) (and subsequently differencing), as well as (23), (29), (30), and (31), yields:

$$\widehat{\pi}_{N,t} = \varsigma_F \widehat{\pi}_{N,t}^f + (1 - \varsigma_F) \widehat{\pi}_{N,t}^b \quad (32)$$

$$\widehat{p}_{N,t} = \widehat{p}_{N,t}^f - \widehat{p}_{N,t}^b \quad (33)$$

$$0 = \varsigma_F \widehat{p}_{N,t}^f + (1 - \varsigma_F) \widehat{p}_{N,t}^b \quad (34)$$

$$\widehat{y}_{N,t}^k = \widehat{y}_{N,t} - \nu \widehat{p}_{N,t}^k \quad (k = f, b) \quad (35)$$

$$\widehat{p}_{N,t} - \widehat{p}_{N,t-1} = \widehat{\pi}_{N,t}^f - \widehat{\pi}_{N,t}^b, \quad (36)$$

where (34) uses the fact (proved further below) that $\bar{p}_N^f = \bar{p}_N^b$. Forward looking firms have a Phillips equation as in the previous subsection:

$$\widehat{\pi}_{N,t}^f = \beta E_t \widehat{\pi}_{N,t+1}^f + \gamma_F \left\{ \widehat{w}_t^f + a_y \left(\widehat{y}_{N,t}^f - \widehat{z}_t^N \right) \right\}, \quad (37)$$

where we defined the product wage for forward looking firms:

$$w_t^f = W_t / P_{N,t}^f.$$

Using (37) and (25) in (32) yields:

$$\begin{aligned} \widehat{\pi}_{N,t} &= \varsigma_F \left\{ \beta E_t \widehat{\pi}_{N,t+1}^f + \gamma_F \left[\widehat{w}_t^f + a_y \left(\widehat{y}_{N,t}^f - \widehat{z}_t^N \right) \right] \right\} + \\ &\quad + (1 - \varsigma_F) \left[\widehat{\pi}_{N,t-1} + \alpha \widehat{p}_{N,t-1} \right] \\ &= \varsigma_F \left\{ \beta E_t \frac{1}{\varsigma_F} \left[\widehat{\pi}_{N,t+1} - (1 - \varsigma_F) \widehat{\pi}_{N,t+1}^b \right] + \gamma_F \left[\widehat{w}_t^f + a_y \left(\widehat{y}_{N,t}^f - \widehat{z}_t^N \right) \right] \right\} + \\ &\quad + (1 - \varsigma_F) \left[\widehat{\pi}_{N,t-1} + \alpha \widehat{p}_{N,t-1} \right] \\ &= \beta E_t \widehat{\pi}_{N,t+1} - (1 - \varsigma_F) \left[\beta \widehat{\pi}_{N,t} - \widehat{\pi}_{N,t-1} \right] - (1 - \varsigma_F) \alpha \left[\beta \widehat{p}_{N,t} - \widehat{p}_{N,t-1} \right] + \\ &\quad + \varsigma_F \gamma_F \left\{ \widehat{w}_t^f + a_y \left(\widehat{y}_{N,t}^f - \widehat{z}_t^N \right) \right\}. \end{aligned}$$

Log-linearizing the definitions of w_t^f and w_t , and using (33)-(36) (which, in particular, imply $\widehat{p}_{N,t}^f = (1 - \varsigma_F)\widehat{p}_{N,t}$), gives:

$$\widehat{w}_t^f = \widehat{W}_t - \widehat{P}_{N,t}^f = \widehat{w}_t - (1 - \varsigma_F)\widehat{p}_{N,t}$$

$$\widehat{y}_{N,t}^f = \widehat{y}_{N,t} - \nu(1 - \varsigma_F)\widehat{p}_{N,t}.$$

Furthermore, note that (36), (32) and (25) imply:

$$\widehat{p}_{N,t} = k\widehat{p}_{N,t-1} + (1/\varsigma_F)(\widehat{\pi}_{N,t} - \widehat{\pi}_{N,t-1}) \quad (38)$$

$$k \equiv 1 - \frac{\alpha}{\varsigma_F}$$

Hence inserting the last three expressions into the previous one yields the following "hybrid" Phillips equation:

$$\widehat{\pi}_{N,t} = h_b\widehat{\pi}_{N,t-1} + h_f E_t \widehat{\pi}_{N,t+1} + h_{mc} \{ \widehat{w}_t + a_y (\widehat{y}_{N,t} - \widehat{z}_t^N) \} + h_p \widehat{p}_{N,t-1} \quad (39)$$

where

$$\begin{aligned} h_b &\equiv \frac{(1 - \varsigma_F)(1 + \Omega)}{(1 - \varsigma_F)(\beta + \Omega) + 1}, & h_f &\equiv \frac{\beta}{(1 - \varsigma_F)(\beta + \Omega) + 1}, \\ h_{mc} &\equiv \frac{\varsigma_F \gamma_F}{(1 - \varsigma_F)(\beta + \Omega) + 1}, & h_p &\equiv \frac{(1 - \varsigma_F)[\alpha(1 + \Omega) - \varsigma_F \Omega]}{(1 - \varsigma_F)(\beta + \Omega) + 1} \\ \Omega &\equiv \frac{\alpha\beta}{\varsigma_F} + \varsigma_F \gamma_F (1 + \nu a_y) > 0. \end{aligned}$$

Note that as ς_F tends to unity (and backward looking firms tend to disappear), the hybrid Phillips equation for non-tradables tends to the purely forward looking one (22). Also, $h_b + h_f$ is in the interval $(\beta, 1)$ as in the hybrid Phillips equations in Svensson (1998), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001), Christiano, Eichenbaum and Evans (2001), Smets and Wouters (2002), and Woodford (2003). Furthermore, the ratio between the forward and backward looking coefficients is

$$\frac{h_f}{h_b} = \frac{\beta}{(1 - \varsigma_F)(1 + \Omega)},$$

which only differs from the ratio in Galí and Gertler (1999) (and in Galí, Gertler and López-Salido (2001)) in having $1 + \Omega$ where they have the inverse of the probability of being able to change price. However, our formulation has the additional term in $\widehat{p}_{N,t-1}$. We will see below that when we eliminate this term there is a substantial change in the restrictions on the remaining coefficients.

Note that only the sign of h_p is ambiguous and essentially depends on the magnitude of α , the "catching up" parameter of backward looking firms. h_p is negative for small values of α , and positive for sufficiently large values of α (taking into account that Ω is a function of α). In the latter case, an increase in the relative price of the non-tradable goods produced by forward looking firms has the effect of increasing non-tradable inflation.

Note that we may use (38) to eliminate $\widehat{p}_{N,t-1}$ from (39), giving a a version of the Phillips equation that is easier to estimate econometrically, and whose parameters are less restricted:

$$\begin{aligned} \widehat{\pi}_{N,t} = & h_{b2}\widehat{\pi}_{N,t-2} + h_{b1}\widehat{\pi}_{N,t-1} + h_{f1}E_t\widehat{\pi}_{N,t+1} + h_{mc1}\{\widehat{w}_t - k\widehat{w}_{t-1} \\ & + a_y[(\widehat{y}_{N,t} - \widehat{z}_t^N) - k(\widehat{y}_{N,t-1} - \widehat{z}_{t-1}^N)]\} + h_\eta\eta_t. \end{aligned} \quad (40)$$

$$\begin{aligned} h_{b2} &\equiv -\frac{(h_p/\varsigma_F) + kh_b}{1 + kh_f}, & h_{b1} &\equiv \frac{(h_p/\varsigma_F) + k + h_b}{1 + kh_f}, \\ h_{f1} &\equiv \frac{h_f}{1 + kh_f}, & h_{mc1} &\equiv \frac{h_{mc}}{1 + kh_f}, \\ h_\eta &\equiv \frac{kh_f}{1 + kh_f}, & \eta_t &\equiv \widehat{\pi}_{N,t} - E_{t-1}\widehat{\pi}_{N,t}. \end{aligned}$$

The coefficients in this version of the Phillips equation are quite different from those in the previous version, since now $h_{b2} + h_{b1} + h_{f1}$ is a function of α/ς_F and can potentially have almost any positive or negative value:

$$h_{b2} + h_{b1} + h_{f1} = \frac{1 + h_f - (1 - h_b)(\alpha/\varsigma_F)}{1 + h_f - h_f(\alpha/\varsigma_F)} \equiv H(\alpha/\varsigma_F).$$

It is easy to prove that $H(\cdot)$ is always decreasing in α/ς_F . For values of α/ς_F lower than $(1 + h_f)/h_f$ (where it has a pole), this function decreases from one to minus infinity, reaching zero at $(1 + h_f)/(1 - h_b)$, and for values larger than that value, $H(\cdot)$ decreases from infinity to $(1 - h_b)/h_f > 1$.

3. Household decisions

We assume that holding money diminishes the cost of transactions in terms of exportable and non-tradable goods.⁹ Let M stand for the nominal stock of currency in circulation, which is the only kind of money considered in this paper. Defining $m_t = M_t/P_t$ and $c_t = C_t/P_t$, the money to consumption ratio is $M_t/C_t = m_t/c_t$. We assume that transactions involve the (non utility generating) consumption of real resources (produced goods) and that these (gross) transaction costs per unit of consumption are a convex function τ of the money/consumption ratio:

$$\tau\left(\frac{m_t}{c_t}\right) \quad (\tau > 1, \tau' < 0, \tau'' > 0). \quad (41)$$

When the money/consumption ratio increases, transaction costs per unit of consumption decrease at a decreasing rate, reflecting a diminishing marginal productivity of money in reducing transaction costs. To obtain private savings we must subtract $\tau(\cdot)c_t$ from income (instead of c_t).

Households are assumed to be monopolistic competitors in the supply of (differentiated) labor. They set the wage rate and face wage adjustment costs. Let $x(\log\pi_{W,t})$ represent the cost per unit of work of changing W_{t-1} at the rate $\pi_{W,t} \equiv$

⁹This way of modeling money demand has been used by Kimbrough (1992), Agénor (1995) and Montiel (1997), among others.

W_t/W_{t-1} . We assume that this adjustment cost function has the same properties as (13) except that $x''(\bar{\pi}_W) = a_H > 0$. We use the same symbol we used for firms' price adjustment cost function for ease of notation.

Below we treat households' budget constraints as if there were a representative household and a single non-tradable good. The model we develop, however, has a continuum of households (each with its labor type) and non-tradable firms (each with its variety of good). We assume that the conditions necessary for all households to face identical budget constraints are satisfied (see Woodford (2003), chapter 3) but without the highly unrealistic complete financial markets assumption. In particular, ownership of forward and backward looking non-tradable sector firms and exportable sector firms are equally distributed among households. In practice, this means that as far as the budget constraints are concerned we can still work with a fictitious "representative household". We do not introduce a subset of backward looking wage setting households in order to avoid having to make even more unrealistic assumptions to ensure household homogeneity or, alternatively, having to deal with the more realistic but cumbersome procedure of keeping a separate accounting for the decision variables of different types of households. We further assume that non-residents do not invest in peso denominated bonds, a typical situation for LDCs that Eichengreen and Hausmann (1999) refer to as "original sin".

Households hold financial net wealth that is composed of domestic money (M_t) and peso denominated (non state contingent) one period nominal bonds issued by the Central Bank (B_t) that pays an interest rate i_t . Hence, their (sequence of) nominal flow budget constraints is given by:

$$M_t + B_t = \Pi_t + W_t^h L_t^h [1 - x(\log(W_t^h/W_{t-1}^h))] - T_t - \tau(M_t/C_t)C_t + M_{t-1} + (1 + i_{t-1})B_{t-1},$$

where Π_t is pre-tax profits, and T_t is lump sum taxes net of transfers. In real terms, the budget constraint is:

$$m_t + b_t = \frac{\Pi_t}{P_t} + \frac{W_t^h}{P_t} L_t^h \left[1 - x \left(\log \left(\frac{W_t^h}{W_{t-1}^h} \right) \right) \right] - t_t - \tau \left(\frac{m_t}{c_t} \right) c_t + \frac{m_{t-1}}{\pi_t} + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} \quad (42)$$

where we defined $\pi_t \equiv P_t/P_{t-1}$, $b_t \equiv B_t/P_t$, and $t_t \equiv T_t/P_t$. The household's inter-temporal solvency is guaranteed by its inability to incur in debt, which we assume does not bind in any finite time:

$$[m_{t+T} + b_{t+T}] \geq 0, \quad \forall T \geq 0. \quad (43)$$

Household $h \in [0,1]$ supplies labor of type h and maximizes an inter-temporal utility function which is additively separable in total consumption of private goods, leisure (or negative work effort), and consumption of public goods:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1 - \sigma} \left(\frac{(c_{X,t+j}^\theta c_{N,t+j}^{1-\theta})}{(c_{X,t+j-1}^\theta c_{N,t+j-1}^{1-\theta})} \right)^\xi \right\}^{1-\sigma} - \frac{v(L_{t+j}^h)}{z_{t+j}^H} + \mathcal{X}(g_{t+j}) \quad (44)$$

where $c_{X,t}$ ($c_{N,t}$) is the consumption of exportable (non-tradable) goods, L_t^h is labor exertion and z_t^H is a labor supply shock which is common to all households (and defined such that a positive shock diminishes the utility of leisure and hence increases labor supply). The consumption part of the instantaneous utility function nests habit formation, where $\xi < 1$ (see Fuhrer (2000)), and Cobb-Douglas sub-utility for exportable and non-tradable goods into a standard constant relative risk aversion (CRRA) utility function, where $\sigma > 0$ is the inverse of the inter-temporal elasticity of substitution (as well as the coefficient of relative risk aversion¹⁰). Consumers hence care about both their level of consumption and their rate of consumption growth. In (44), the function $v(\cdot)$ represents the disutility of labor, which is assumed to be increasing and convex ($v' > 0$, $v'' > 0$), and $\varkappa(g_t)$ represents the utility obtained by the household from the quantities of public goods produced by the government (which is a function of the quantities purchased by the government). Since g_t is not a decision variable for the household, $\varkappa(\cdot)$ does not play a significant role except when the evaluation of alternative fiscal policies comes into play.

In analogy to the ‘employment agency’, we assume that there is a competitive ‘commercial agency’ (or ‘representative consumption aggregator’) that combines the different non-tradable goods into a single bundle, in the proportions dictated by households’ preferences. The commercial agency’s composite $c_{N,t}$ is defined by:¹¹

$$c_{N,t} = \left\{ \int_0^1 (c_{N,i,t})^{(\nu-1)/\nu} di \right\}^{\nu/(\nu-1)} \quad (\nu > 1). \quad (45)$$

For any level of the composite $c_{N,t}$ the agency minimizes expenditures, given the prices $P_{N,i,t}$ set by the individual firms. Hence, it minimizes

$$\int_0^1 P_{N,i,t} c_{N,i,t} di$$

subject to (45) for a given value of $c_{N,t}$. This gives total consumption demand for $c_{N,i,t}$:

$$c_{N,i,t} = (P_{N,i,t}/P_{N,t})^{-\nu} c_{N,t}, \quad (46)$$

where $P_{N,t}$ is given by (15). Furthermore, total expenditure on non-tradables is

$$\int_0^1 P_{N,i,t} c_{N,i,t} di = P_{N,t} c_{N,t}.$$

Hence, total real consumption expenditure is:

$$c_t = \frac{C_t}{P_t} = \frac{1}{P_t} \left[\left(\frac{\phi_t S_t}{\rho_t} \right) c_{X,t} + P_{N,t} c_{N,t} \right] = e_t^{1-\theta} \left(c_{X,t} + \frac{c_{N,t}}{e_t} \right). \quad (47)$$

Minimizing the r.h.s. of the last equality subject to a constant (and arbitrary) level of sub-utility $(c_{X,t})^\theta (c_{N,t})^{1-\theta}$ gives:

$$\frac{e_t c_{X,t}}{c_{N,t}} = \frac{\theta}{1-\theta}.$$

¹⁰If $u(c) \equiv c^{1-\sigma}/(1-\sigma)$, the coefficient of relative risk aversion is $-cu''(c)/u'(c) = \sigma$.

¹¹Alternatively (and equivalently), we can assume that non tradable goods are intermediate goods and that final goods producing firms are perfectly competitive and the representative firm has (45) as its production function.

Note that the last two expressions imply

$$c_{N,t} = (1 - \theta)e_t^\theta c_t, \quad c_{X,t} = \theta e_t^{-(1-\theta)} c_t, \quad (48)$$

$$c_{X,t}^\theta c_{N,t}^{1-\theta} = \kappa_0 c_t \quad (\kappa_0 \equiv \theta^\theta (1-\theta)^{1-\theta}).$$

The first two equalities in (48) show that consumption demands for N and X are easily obtained from c and e , so henceforth we work with the latter two variables. Inserting the third equality of (48) in (44) gives:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\kappa_1}{1-\sigma} \left(\frac{c_{t+j}}{(c_{t+j-1})^\xi} \right)^{1-\sigma} - \frac{v(L_{t+j}^h)}{z_{t+j}^H} + \varkappa(g_{t+j}) \right\}, \quad (49)$$

$$\left(\kappa_1 \equiv \kappa_0^{(1-\sigma)(1-\xi)} \right).$$

Household h chooses c_t, m_t, b_t , and W_t^h , to maximize (49) subject to its sequence of budget constraints, its labor demand function (10), and its “no debt” condition (43) for all t . The Lagrangian is:

$$E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{\kappa_1}{1-\sigma} \left(\frac{c_{t+j}}{(c_{t+j-1})^\xi} \right)^{1-\sigma} - v \left(L_{t+j} \left(\frac{W_{t+j}^h}{W_{t+j}} \right)^{-\psi} \right) \frac{1}{z_{t+j}^H} \right. \quad (50)$$

$$+ \varkappa(g_{t+j}) + \lambda_{t+j} \left\{ \frac{\Pi_{t+j}}{P_{t+j}} + \frac{W_{t+j}^h}{P_{t+j}} L_{t+j} \left(\frac{W_{t+j}^h}{W_{t+j}} \right)^{-\psi} \left[1 - x \left(\log \left(\frac{W_{t+j}^h}{W_{t-1+j}^h} \right) \right) \right] \right.$$

$$\left. \left. - t_{t+j} - \tau \left(\frac{m_{t+j}}{c_{t+j}} \right) c_{t+j} + \frac{m_{t-1+j}}{\pi_{t+j}} + (1 + i_{t-1+j}) \frac{b_{t-1+j}}{\pi_{t+j}} - m_{t+j} - b_{t+j} \right\} \right\}$$

where λ_{t+j} are the Lagrangian multipliers, which can be interpreted as the marginal utilities of real income. The first order conditions for an optimum (including the transversality condition) are the following:

$$c_t : \quad \frac{\kappa_1}{c_t} \left[\left(\frac{c_t}{(c_{t-1})^\xi} \right)^{1-\sigma} - \beta \xi E_t \left(\frac{c_{t+1}}{(c_t)^\xi} \right)^{1-\sigma} \right] = \lambda_t \varphi \left(\frac{m_t}{c_t} \right) \quad (51a)$$

$$m_t : \quad 1 + \tau'(m_t/c_t) = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left/ \frac{P_{t+1}}{P_t} \right. \right) \quad (51b)$$

$$b_t : \quad 1 = \beta(1 + i_t) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left/ \frac{P_{t+1}}{P_t} \right. \right) \quad (51c)$$

$$W_t^h : \quad G_t^W = \mu_H \frac{e_t^\theta}{\lambda_t w_t z_t^H} v'(L_t), \quad \left(\mu_H \equiv \frac{\psi}{\psi - 1} \right) \quad (51d)$$

$$\lim_{t \rightarrow \infty} \beta^t \{m_t + b_t\} = 0. \quad (52)$$

We have used the auxiliary functions $\varphi(\cdot)$ and G_t^W that we define below.

The wage inflation Phillips equation

The (inverse) wage markup gap G_t^W in (51d) is defined in complete analogy to the non-tradable firms' markup gap (20):

$$G_t^W \equiv 1 - x(\log \pi_{W,t}) + \frac{1}{\psi - 1} \left\{ x'(\log \pi_{W,t}) - E_t \left[\beta \left[\frac{L_{t+1}}{L_t} \frac{w_{t+1}}{w_t} / \left(\frac{e_{t+1}}{e_t} \right)^\theta \right] x'(\log \pi_{W,t+1}) \right] \right\}. \quad (53)$$

μ_H is the monopolistic competition markup over the marginal rate of substitution of real income for leisure when all wages (and prices) are flexible (as in the non-stochastic steady state). Since all households face the same problem, they all set the same wage, so we have dropped h from (51d) and (53). In analogy to the case of price setting firms, in the steady state with zero inflation the wage gap collapses to unity, which implies that the wage is a constant markup over the marginal rate of substitution of real income for leisure:

$$w^\circ = \mu_H \frac{v'(\bar{L})}{\lambda}. \quad (54)$$

In this expression we have used the definition of the real wage (6).

Log-linearizing (51d) and (53) (see the Appendix for the similar case of non-tradable inflation) yields a wage inflation "Phillips curve" equation:

$$\widehat{\pi}_{W,t} = \beta E_t \widehat{\pi}_{W,t+1} + \gamma_H \{ \theta \widehat{e}_t - \widehat{w}_t - \widehat{\lambda}_t + a_L \widehat{L}_t - \widehat{z}_t^H \}, \quad (55)$$

$$\gamma_H \equiv \frac{\psi - 1}{a_H}, \quad a_L \equiv \bar{\varepsilon}_{v'} = \frac{v''(\bar{L})\bar{L}}{v'(\bar{L})}.$$

Money and consumption demand

To alleviate notation, in (51a) we have defined the auxiliary function φ that gives the reduction in savings due to a marginal increase in consumption:¹²

$$\varphi \left(\frac{m_t}{c_t} \right) \equiv \tau \left(\frac{m_t}{c_t} \right) - \left(\frac{m_t}{c_t} \right) \tau' \left(\frac{m_t}{c_t} \right), \quad (56)$$

$$\varphi' \left(\frac{m_t}{c_t} \right) = - \left(\frac{m_t}{c_t} \right) \tau'' \left(\frac{m_t}{c_t} \right) < 0.$$

Observe that φ is decreasing in m_t/c_t and that the reduction in savings generated by a marginal increase in c_t is given by the reduction in savings with the initial money/consumption ratio, τ , plus the increase in transaction costs due to the reduction in the money/consumption ratio, $(m_t/c_t)(-\tau')$.

(51a) shows that in equilibrium the marginal utility of consumption (left side of the equality) must be equal to the marginal disutility of the reduction in real income that it generates. The latter is equal to the marginal utility of real income, λ_t , times the marginal reduction in savings, $\varphi(\cdot)$.

¹² $\varphi(m/c)$ is the partial derivative of $\tau(m/c)c$ with respect to c . Hence, $-\varphi(\cdot)$ is the effect of a marginal increase in c on savings.

Combining (51b) and (51c) yields:

$$-\tau' \left(\frac{m_t}{c_t} \right) = 1 - \frac{1}{1 + i_t}, \quad (57)$$

which shows that in the optimum money holdings must be such that the reduction in transaction costs generated by a marginal increase in money holdings equals the opportunity cost of holding money. Inverting $-\tau'$ gives the following demand function for money as a vehicle for transactions (sometimes called "liquidity preference" function):

$$m_t = (-\tau')^{-1} \left(1 - \frac{1}{1 + i_t} \right) c_t \equiv \ell(1 + i_t) c_t, \quad (58)$$

$$\ell'(1 + i_t) = \frac{1}{-\tau''(1 + i_t)^2} < 0.$$

Inserting this expression in (51a) (as we do below in (63)) yields a complex relation between c_t , λ_t , and i_t , which defines the dynamics of household consumption demand.

Furthermore, log-linearizing (51c) yields:

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{i}_t - E_t \widehat{\pi}_{t+1}, \quad (59)$$

which states that the expected change in the marginal utility of real income is equal to minus the expected real interest rate (all in log deviations from steady state values).

4. Clearing in the money, domestic currency bonds, non-tradable goods, and labor markets

We assume that the Central Bank always provides a supply of money and bonds that matches demand. Hence, in the case of money (58) yields the equilibrium stock of real money as a function of c_t , and $1 + i_t$:

$$m_t = \ell(1 + i_t) c_t. \quad (60)$$

This is a key equation for the monetary aspects of the model. The log-linear approximation to (60) is:

$$\widehat{m}_t = \widehat{c}_t - \bar{\varepsilon}_\ell \widehat{i}_t, \quad \bar{\varepsilon}_\ell \equiv \frac{-\ell'(\bar{\pi}/\beta)(\bar{\pi}/\beta)}{\ell(\bar{\pi}/\beta)}, \quad (61)$$

where we use the fact (directly implied by (51c)) that $1 + \bar{i} = \bar{\pi}/\beta$. Similarly, by using the same notation for private sector demand for Central Bank bonds (as in the household's budget constraint) and for Central Bank supply (which we do in the next section), we implicitly assume that this market clears every period.

Let us define the following auxiliary functions

$$\bar{\tau}(1 + i_t) \equiv \tau(\ell(1 + i_t)), \quad \bar{\varphi}(1 + i_t) \equiv \varphi(\ell(1 + i_t)), \quad (62)$$

both of which are increasing in their argument. Inserting (60) in (51a) yields the following complex relation between consumption demand, the marginal utility of real income, and the nominal interest rate:

$$\left(\frac{c_t}{(c_{t-1})^\xi}\right)^{1-\sigma} - \beta\xi E_t \left(\frac{c_{t+1}}{(c_t)^\xi}\right)^{1-\sigma} = \frac{c_t \lambda_t \bar{\varphi}(1+i_t)}{\kappa_1}, \quad (63)$$

the log-linear version of which is:

$$\widehat{c}_t = a_0 \widehat{c}_{t-1} + a_1 E_t \widehat{c}_{t+1} - a_2 \widehat{\lambda}_t - a_3 \widehat{i}_t, \quad (64)$$

$$\begin{aligned} a_0 &\equiv \frac{(\sigma-1)\xi\kappa_1}{(\sigma-1)(1+\beta\xi^2)\kappa_1 + (1-\beta\xi)}, & a_1 &\equiv \beta a_0, \\ a_2 &\equiv \frac{1-\beta\xi}{(\sigma-1)(1+\beta\xi^2)\kappa_1 + (1-\beta\xi)}, & a_3 &\equiv a_2 \bar{\varepsilon}_\varphi = a_2 \frac{\bar{\varphi}'(\bar{\pi}/\beta)(\bar{\pi}/\beta)}{\bar{\varphi}(\bar{\pi}/\beta)}. \end{aligned}$$

The size of the relative risk aversion coefficient σ is an empirical matter. For quarterly data, it is usually measured as much greater than one. We merely assume $\sigma > 1$, which makes all the a_i positive since both β and ξ are less than unity.

To simplify, assume that government expenditure on each type of good is the same fraction g_t^* of private consumption demand for that good (inclusive of transaction costs):

$$g_{X,t} = g_t^* \bar{\tau}(1+i_t) c_{X,t}, \quad g_{N,i,t} = g_t^* \bar{\tau}(1+i_t) c_{N,i,t}. \quad (65)$$

Hence, to clear the non-tradables market total output of type i must be:

$$y_{N,i,t} = \frac{(1+g_t^*)\bar{\tau}(1+i_t)c_{N,i,t}}{1-x(\log\pi_{N,i,t}^f)} = \frac{(1+g_t^*)\bar{\tau}(1+i_t)c_{N,i,t}}{1-x(\log\pi_{N,t}^f)}, \quad i \in [0, \varsigma_F] \quad (66)$$

$$y_{N,i,t} = (1+g_t^*)\bar{\tau}(1+i_t)c_{N,i,t}, \quad i \in (\varsigma_F, 1],$$

where non-tradable consumption demand for optimizing firms must be grossed up to include the real resources used up in the price adjustment decision process. The second equality in the first line of (66) stems from the fact that all optimizing firms in a symmetric equilibrium face the same price adjustment costs. Since all firms within the same category have the same decision process, the use of (66) and (46) yields the non-tradables demand functions (14) used in section 2. Aggregating over non-tradable types as in (45) and using (48) gives the market clearing bundle of non-tradable output:

$$\begin{aligned} y_{N,t} &= \varsigma_F [(1+g_t^*)\bar{\tau}(1+i_t)(1-\theta)e_t^\theta c_t] \frac{1}{1-x(\log\pi_{N,t}^f)} + \\ &\quad + (1-\varsigma_F) [(1+g_t^*)\bar{\tau}(1+i_t)(1-\theta)e_t^\theta c_t] \\ &= [(1+g_t^*)\bar{\tau}(1+i_t)(1-\theta)e_t^\theta c_t] \left[1 + \varsigma_F \frac{x(\log\pi_{N,t}^f)}{1-x(\log\pi_{N,t}^f)} \right]. \end{aligned} \quad (67)$$

Note that, for a given level of consumption, the influence of the nominal interest rate on non-tradable output is positive because a higher interest rate makes

households economize on money holdings, which implies a greater use of resources (non-tradable goods) in transactions. However, we have seen in (63) and (64) that there is also a negative effect of the interest rate on consumption, and hence on output which, to be realistic, should predominate. On the other hand, the effect of the MRER on non-tradable output is positive because a real depreciation makes exportable goods relatively more expensive and thus shifts consumption demand towards non-tradables.

We now derive total labor requirements. (7) gives non tradable firm i 's demand for labor as $L_{N,i,t} = F_N^{-1}(y_{N,i,t}/z_t^N)$. Since all non-tradable forward looking (backward looking) firms produce the same amount (of their specific type of goods), they all produce $y_{N,t}^f$ ($y_{N,t}^b$) using the same amount of the labor input bundle L_N^f (L_N^b). Therefore, labor demand in the non-tradable sector is:

$$L_{N,t} = \varsigma_F F_N^{-1} \left(\frac{y_{N,t}^f}{z_t^N} \right) + (1 - \varsigma_F) F_N^{-1} \left(\frac{y_{N,t}^b}{z_t^N} \right),$$

and, using (12) to obtain labor demand by the export sector, the labor market clearing condition is:

$$L_t = \left[(F_X')^{-1} \left(\frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t} \right) + \varsigma_F F_N^{-1} \left(\frac{y_{N,t}^f}{z_t^N} \right) + (1 - \varsigma_F) F_N^{-1} \left(\frac{y_{N,t}^b}{z_t^N} \right) \right] \frac{1}{1 - x(\log \pi_{W,t})} \quad (68)$$

where the r.h.s. of this equality represents total labor requirements (including labor used in wage adjustment decisions.

The log-linearized versions of the two resource constraints (67) and (68) are:

$$\hat{y}_{N,t} = \hat{c}_t + \theta \hat{e}_t + a_I \hat{u}_t + \hat{g}_t \quad (69)$$

$$\hat{L}_t = a_{LX} \left[\hat{e}_t - \hat{w}_t + \beta_\phi (\hat{\phi}_t - \hat{\epsilon}_t) \right] + a_{LN} \hat{y}_{N,t} + a_{LX} \hat{z}_t^X - a_{LN} \hat{z}_t^N \quad (70)$$

$$\begin{aligned} \hat{g}_t &\equiv \log \left(\frac{1 + g_t^*}{1 + \bar{g}^*} \right), & a_I &\equiv \bar{\epsilon}_\tau = \frac{\bar{\tau}'(\bar{\pi}/\beta)(\bar{\pi}/\beta)}{\bar{\tau}(\bar{\pi}/\beta)}, \\ \beta_\phi &\equiv \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}}, & a_{LN} &\equiv \frac{1}{\bar{\epsilon}_{F_N}} \frac{\bar{L}_N}{\bar{L}} = \frac{F_N(\bar{L}_N)}{F_N'(\bar{L}_N) \bar{L}_N} \frac{\bar{L}_N}{\bar{L}}, \\ a_{LX} &\equiv \frac{1}{\bar{\epsilon}_{F_X'}} \frac{\bar{L}_X}{\bar{L}} = \frac{F_X'(\bar{L}_X)}{F_X''(\bar{L}_X) F_X(\bar{L}_X)} \frac{\bar{L}_X}{\bar{L}}. \end{aligned}$$

5. The public sector and the balance of payments

The public sector is made up of the Government and the Central Bank. The Government issues dollar denominated bonds in the international markets, spends on tradables and non-tradables, and collects taxes, while the Central Bank issues money and domestic currency bonds (only held by residents) and holds international reserves R_t . We assume that fiscal policy consists of exogenous paths for

tax collection (t_t) and expenditures (g_t) and debt financing for any deficit by issuing dollar denominated bonds. These paths are assumed to be compatible with a finite non-stochastic steady state for government debt. To hold foreign currency denominated government bonds, international investors charge a risk premium (p_t) over the risk-free dollar interest rate (r_t^*). Since we do not model the rest of the world, the risk premium (function) is exogenously given and is assumed to have an exogenous stochastic component (an external financing shock) and an endogenous component which is an increasing function of the government's (and country's) net foreign liabilities, i.e. the stock of government bonds net of Central Bank international reserves.¹³

$$p_t \equiv \zeta_t + p(D_t - R_t), \quad p' > 0.$$

Hence, international investors' portfolio decision makes them willing to invest in risky SOE Government bonds only if the interest rate i_t^* satisfies:

$$1 + i_t^* = (1 + r_t^*)[1 + \zeta_t + p(D_t - R_t)] \quad (71)$$

We also assume that there are arbitrageurs who make use of any profit opportunities, and hence ensure that the uncovered interest parity condition (UIP) between domestic peso and dollar bonds is satisfied:¹⁴

$$1 + i_t = (1 + i_t^*) E_t \delta_{t+1} \quad (72)$$

where we have defined the rate of nominal depreciation of the peso against the dollar $\delta_t = S_t/S_{t-1}$. The log-linear versions of the last two equations are:

$$\begin{aligned} \widehat{i}_t^* &= \widehat{r}_t^* + \alpha_\zeta \widehat{\zeta}_t + (1 - \alpha_\zeta) \bar{\varepsilon}_p \left[\alpha_G \widehat{D}_t - (1 - \alpha_G) \widehat{R}_t \right] \\ \widehat{i}_t &= \widehat{i}_t^* + E_t \widehat{\delta}_{t+1}, \end{aligned} \quad (73)$$

$$\begin{aligned} \bar{\varepsilon}_p &\equiv \frac{p'(\bar{D} - \bar{R})(\bar{D} - \bar{R})}{p(\bar{D} - \bar{R})}, & \alpha_G &\equiv \frac{\bar{D}}{\bar{D} - \bar{R}} \\ \alpha_\zeta &\equiv \frac{1 + \bar{\zeta}}{1 + \bar{\zeta} + p(\bar{D} - \bar{R})}. \end{aligned}$$

Note that in the case of interest rates (and government expenditures) our notation differs slightly since we define $\widehat{i}_t = \log [(1 + i_t)/(1 + \bar{i})]$, and similarly for other interest rates.

¹³A more general formulation would have R_t as a separate argument in $p(\cdot)$, with $p_R < 0$, reflecting the negative effect of larger amounts of international reserves on liquidity risk. But this formulation would still present a negative effect of reserves on $p(\cdot)$. Hence, we leave the simpler formulation.

¹⁴In section 8 we approach a more conventional view of the workings of Central Bank sterilized intervention in the foreign exchange market by assuming that households also view Central Bank peso bonds as risky, which introduces a risk premium that is increasing in b_t in (72).

Some national accounting will help in simplifying the household's budget constraint. Using (8) and (67), aggregate nominal output (net of imports and resources used in price setting) is:

$$Y_t = (S_t/\rho_t)(\phi_t - \epsilon_t)y_{X,t} + P_{N,t}y_{N,t} \left/ \left[1 + \varsigma_F \frac{x(\log \pi_{N,t}^f)}{1 - x(\log \pi_{N,t}^f)} \right] \right.$$

Then, using (5), the real value of aggregate output $Y_t/P_t \equiv y_t$ is:

$$y_t = e_t^{1-\theta} \left(1 - \frac{\epsilon_t}{\phi_t} \right) y_{X,t} + e_t^{-\theta} y_{N,t} \left/ \left[1 + \varsigma_F \frac{x(\log \pi_{N,t}^f)}{1 - x(\log \pi_{N,t}^f)} \right] \right. . \quad (74)$$

Therefore, using the labor resource constraint (68), in equilibrium aggregate real profit is:

$$\begin{aligned} \frac{\Pi_t}{P_t} &= \left\{ e_t^{1-\theta} \left(1 - \frac{\epsilon_t}{\phi_t} \right) y_{X,t} - \frac{W_t}{P_t} L_{X,t} \right\} + \\ &\quad + \left\{ e_t^{-\theta} y_{N,t} \left/ \left[1 + \varsigma_F \frac{x(\log \pi_{N,t})}{1 - x(\log \pi_{N,t})} \right] \right. - \frac{W_t}{P_t} L_{N,t} \right\} \\ &= y_t - \frac{W_t}{P_t} L_t [1 - x(\log \pi_{W,t})]. \end{aligned}$$

We can use this expression to simplify the household budget constraint (42) to (76). Hence, the real non-interest (and pre-tax) income (net of the value of labor used in the wage setting process) of households is:

$$y_t = \frac{\Pi_t}{P_t} + \frac{W_t}{P_t} L_t [1 - x(\log \pi_{W,t})], \quad (75)$$

and the (flow) budget constraints of households, the Government and the Central Bank are:

$$m_t + b_t = y_t - t_t - \bar{\tau}(1 + i_t)c_t + \frac{m_{t-1}}{\pi_t} + (1 + i_{t-1})\frac{b_{t-1}}{\pi_t} \quad (76)$$

$$s_t D_t = g_t - t_t + (1 + i_{t-1}^*)s_t D_{t-1} \quad (77)$$

$$m_t + b_t - s_t R_t = \frac{m_{t-1}}{\pi_t} + (1 + i_{t-1})\frac{b_{t-1}}{\pi_t} - (1 + r_{t-1}^*)s_t R_{t-1} \quad (78)$$

where $s_t \equiv S_t/P_t$.

We assume that the Central Bank has the policy of maintaining a full backing of its peso liabilities with international reserves in every period. It does so by transferring its real quasi-fiscal surplus or deficit to the Government every period. This includes all the factors that would otherwise change the net worth of the Central Bank:

$$\begin{aligned} \left[\frac{m_t}{\pi_{t+1}} - \frac{m_{t-1}}{\pi_t} \right] + \left[(1 + i_t)\frac{b_t}{\pi_{t+1}} - (1 + i_{t-1})\frac{b_{t-1}}{\pi_t} \right] - \\ - \left[(1 + r_t^*)s_{t+1}R_t - (1 + r_{t-1}^*)s_t R_{t-1} \right]. \end{aligned}$$

Therefore, the Central Bank's balance sheet always shows international reserves that are equal in real value to its real liabilities:¹⁵

$$m_t + b_t = s_t R_t. \quad (79)$$

In our model, this equation implicitly defines the Central Bank's supply of peso denominated bonds, given the other variables. As in the case of money, while the household budget constraint gives households' demand for Central Bank bonds, our use of the same symbol for Central Bank supply means that we are assuming that this market clears every period. The log-linear version of this equation is:

$$\widehat{R}_t = \alpha_m \widehat{m}_t + (1 - \alpha_m) \widehat{b}_t - \widehat{s}_t, \quad \alpha_m \equiv \frac{\bar{m}}{\bar{m} + \bar{b}}.$$

Adding (77) and (78) gives the consolidated public sector budget constraint:

$$\begin{aligned} m_t + b_t + s_t (D_t - R_t) &= g_t - t_t + \frac{m_{t-1}}{\pi_t} + (1 + i_{t-1}) \frac{b_{t-1}}{\pi_t} + \\ &+ (1 + i_{t-1}^*) s_t D_{t-1} - (1 + r_{t-1}^*) s_t R_{t-1}. \end{aligned} \quad (80)$$

And subtracting (76) from this equation and using (65) and (45), yields the balance of payments equation:

$$\begin{aligned} D_t - R_t &= \frac{1}{s_t} [(1 + g_t^*) \bar{\tau} (1 + i_t) c_t - y_t] + \\ &+ (1 + i_{t-1}^*) D_{t-1} - (1 + r_{t-1}^*) R_{t-1}. \end{aligned} \quad (81)$$

The first term in the r.h.s. of this equation is the trade deficit ($-TB_t$). We can eliminate non-tradables by using (47), (65), (74), (48), and (5) and hence define the trade balance as:

$$\begin{aligned} TB_t &= \left(\frac{\phi_t - \epsilon_t}{\rho_t} \right) y_{X,t} - \frac{1}{s_t} [(1 + g_t^*) \bar{\tau} (1 + i_t) \theta c_t] \\ y_{X,t} &\equiv z_t^X F_X \left((F_X')^{-1} \left(\frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t} \right) \right). \end{aligned}$$

Therefore, the balance of payments equation (81) can be written as:

$$D_t - R_t = -TB_t + (1 + i_{t-1}^*) D_{t-1} - (1 + r_{t-1}^*) R_{t-1}. \quad (82)$$

The log-linear versions of the balance of payments equations and the expressions for the trade balance, net exports and aggregate output (74) are:

$$\begin{aligned} -\widehat{TB}_t &= \alpha_D \left\{ \widehat{D}_t - (\bar{\pi}^*/\beta) \widehat{D}_{t-1} - (\bar{\pi}^*/\beta) \widehat{i}_{t-1}^* \right\} - \\ &- \alpha_R \left\{ \widehat{R}_t - (1 + \bar{r}^*) \widehat{R}_{t-1} - (1 + \bar{r}^*) \widehat{r}_{t-1}^* \right\} \end{aligned}$$

¹⁵We assume that the full backing policy began in some period T in which there was full backing. Hence, the assumption on the quasi-deficit ensures that (79) holds for all t.

$$\begin{aligned}
\widehat{TB}_t &= \widehat{\phi}_t - \widehat{\rho}_t + \beta_X \left[\widehat{y}_{X,t} + (\beta_\phi - 1) (\widehat{\phi}_t - \widehat{\epsilon}_t) \right] - \\
&\quad - (\beta_X - 1) \left[\widehat{c}_t + \widehat{g}_t^* - (1 - \theta) \widehat{e}_t + a_I \widehat{i}_t \right] \\
\widehat{y}_{X,t} &= \varepsilon_{FF} \left[\widehat{e}_t - \widehat{w}_t + \beta_\phi (\widehat{\phi}_t - \widehat{\epsilon}_t) \right] + (1 + \varepsilon_{FF}) \widehat{z}_t^X \\
\widehat{y}_t &= (1 - \beta_y) \widehat{y}_{N,t} + \beta_y \left[\widehat{y}_{X,t} + (\beta_\phi - 1) (\widehat{\phi}_t - \widehat{\epsilon}_t) \right] + (\beta_y - \theta) \widehat{e}_t,
\end{aligned}$$

where:

$$\begin{aligned}
\beta_X &\equiv \frac{(1 - \bar{\epsilon}/\bar{\phi}) \bar{y}_X}{(1 - \bar{\epsilon}/\bar{\phi}) \bar{y}_X - (1 + \bar{g}^*) \bar{\tau} (\bar{\pi}/\beta) \theta \bar{c}/\bar{e}^{1-\theta}}, & \varepsilon_{FF} &\equiv \frac{\bar{\epsilon}_{Fx}}{\bar{\epsilon}_{F'x}}, \\
\alpha_D &\equiv \frac{\bar{D}}{(\bar{\pi}/\beta - 1) \bar{D} - \bar{r}^* \bar{R}}, & \alpha_R &\equiv \frac{\bar{R}}{(\bar{\pi}/\beta - 1) \bar{D} - \bar{r}^* \bar{R}}, \\
\beta_y &\equiv \frac{(1 - \bar{\epsilon}/\bar{\phi}) \bar{y}_X}{(1 - \bar{\epsilon}/\bar{\phi}) \bar{y}_X + \bar{y}_N/\bar{e}}.
\end{aligned}$$

6. The non-linear systems when there are only forward looking firms

For the reader's convenience, we first gather the equations common to the non linear systems assuming that all firms optimize ($\zeta_F = 1$) and leaving out the equations related to monetary and exchange rate policy. We then close make alternative closures of the system with the Central Bank policy equations. Since our purpose here and in the next section is to ensure model consistency and derive the non-stochastic steady state, we momentarily leave out the backward looking firms. In section 8 we show how the introduction of "rule of thumb" firms modifies the non-linear model without affecting the non-stochastic steady state. We first gather the following 20 equations:

$$G_t^P = \mu_F \frac{w_t}{F_N'(F_N^{-1}(y_{N,t}/z_t^N))} \quad (83)$$

$$G_t^W = \mu_H \frac{e_t^\theta}{\lambda_t w_t z_t^H} v'(L_t) \quad (84)$$

$$\left(\frac{c_t}{(c_{t-1})^\xi} \right)^{1-\sigma} - \beta \xi E_t \left(\frac{c_{t+1}}{(c_t)^\xi} \right)^{1-\sigma} = \lambda_t \varphi \left(\frac{m_t}{c_t} \right) \frac{c_t}{\kappa_1} \quad (85)$$

$$1 + \tau' \left(\frac{m_t}{c_t} \right) = \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} / \pi_{t+1} \right) \quad (86)$$

$$1 = \beta(1 + i_t) E_t \left(\frac{\lambda_{t+1}}{\lambda_t} / \pi_{t+1} \right) \quad (87)$$

$$1 + i_t = (1 + i_t^*) E_t \delta_{t+1} \quad (88)$$

$$1 + i_t^* = (1 + r_t^*) [1 + \zeta_t + p(D_t - R_t)] \quad (89)$$

$$y_{N,t} = \left[(1 + g_t^*) \tau \left(\frac{m_t}{c_t} \right) (1 - \theta) e_t^\theta c_t \right] \frac{1}{1 - x(\log \pi_{N,t})} \quad (90)$$

$$L_t \equiv \left[(F_X')^{-1} \left(\frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t} \right) + F_N^{-1} \left(\frac{y_{N,t}}{z_t^N} \right) \right] \frac{1}{1 - x(\log \pi_{W,t})} \quad (91)$$

$$m_t + b_t = s_t R_t \quad (92)$$

$$D_t - R_t = -TB_t + (1 + i_{t-1}^*) D_{t-1} - (1 + r_{t-1}^*) R_{t-1} \quad (93)$$

$$\delta_t = \pi_{N,t} \frac{e_t}{e_{t-1}} \frac{\rho_t}{\rho_{t-1}} \bigg/ \frac{\phi_t}{\phi_{t-1}} \quad (94)$$

$$\pi_t = \left(\frac{e_t}{e_{t-1}} \right)^\theta \pi_{N,t} \quad (95)$$

$$w_t = w_{t-1} \frac{\pi_{W,t}}{\pi_{N,t}} \quad (96)$$

$$s_t = \frac{\rho_t}{\phi_t} e_t^{1-\theta} \quad (97)$$

$$G_t^P = 1 - x(\log \pi_{N,t}) + \frac{1}{\nu - 1} \quad (98)$$

$$\left\{ x'(\log \pi_{N,t}) - E_t \left[\frac{1}{1 + i_t} \frac{y_{N,t+1}}{y_{N,t}} \pi_{N,t+1} x'(\log \pi_{N,t+1}) \right] \right\}$$

$$G_t^W = 1 - x(\log \pi_{W,t}) + \frac{1}{\psi - 1} \quad (99)$$

$$\left\{ x'(\log \pi_{W,t}) - E_t \left[\beta \left(\frac{L_{t+1}}{L_t} \frac{w_{t+1}}{w_t} \bigg/ \left(\frac{e_{t+1}}{e_t} \right)^\theta \right) x'(\log \pi_{W,t+1}) \right] \right\}$$

$$y_{X,t} = z_t^X F_X \left((F_X')^{-1} \left(\frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t} \right) \right) \quad (100)$$

$$y_t = e_t^{1-\theta} \left(1 - \frac{\epsilon_t}{\phi_t} \right) y_{X,t} + \frac{y_{N,t}}{e_t^\theta} [1 - x(\log \pi_{N,t})] \quad (101)$$

$$TB_t = \left(\frac{\phi_t - \epsilon_t}{\rho_t} \right) y_{X,t} - \frac{1}{s_t} (1 + g_t^*) \tau \left(\frac{m_t}{c_t} \right) \theta c_t \quad (102)$$

These 20 equations are insufficient to determine the paths of the 22 endogenous variables: $\pi_{N,t}$, $\pi_{W,t}$, π_t , δ_t , e_t , w_t , c_t , λ_t , $y_{N,t}$, $y_{X,t}$, y_t , L_t , i_t , i_t^* , m_t , b_t , D_t , G_t^P , G_t^W , R_t , s_t , TB_t . We will shortly close the system with 2 Central Bank policy equations under alternative regimes. But let us first recapitulate what these equations represent. The first equation is the optimizing firm's first order condition. Equations (84) to (87) are the household's first order conditions. Equation (88) is the UIP condition. Next, equation (89) shows the formation of the dollar interest rate as compounding the international dollar riskless rate and international investors' risk premium. Equations (90) and (91) are the market clearing conditions for non-tradable goods and labor, respectively.¹⁶ Equations (92) and (93) are the Central

¹⁶We remind the reader that by not making a distinction between household demand for money and peso bonds (say m_t^d and b_t^d) and Central Bank supply for these bonds (say m_t and b_t) we do not include explicit market clearing conditions for these assets ($m_t^d = m_t$ and $b_t^d = b_t$). Hence m_t and b_t denote the market clearing stocks of these assets.

Bank full backing condition and the balance of payments equation, respectively. Equations (94) to (96) are identities derived from the definitions of the variables involved. And equations (98) to (102) are simply definitions of auxiliary variables that have been used to make the preceding equations simpler to read. We now consider alternative regimes.

6.1 Fixed exchange rate regimes

When the Central Bank fixes the exchange rate it abstains from actively intervening in the money market. Hence, we assume that it maintains its real liabilities in domestic currency bonds constant:

$$b_t = b_0 \quad \forall t. \quad (103)$$

6.2 A unilaterally fixed exchange rate or rate or crawl (UFIX)

Let us first assume that the Central Bank pegs the nominal exchange rate to the currency of one of its trade partners, say the dollar, by intervening in the foreign exchange market so as to ensure that the rate of nominal depreciation follows a predetermined path $\{\delta_t^*\}$, that is such that $S_t/S_{t-1} = \delta_t^*$, for all t . We may formalize the feedback rule as the limit of a "leaning against the wind" policy where the Central Bank counteracts (excessive) nominal appreciations (depreciations) by purchasing (selling) international reserves. In the limit, the Central Bank counteracts any deviation whatsoever of the rate of nominal depreciation from its target level (which is 1 in the case of a fixed exchange rate and is constant at some level $\bar{\delta}^*$ in the case of a fixed crawl):

$$R_t = \lim_{k_1 \rightarrow \infty} R_{t-1} \left(\frac{S_t/S_{t-1}}{\delta_t^*} \right)^{-k_1}. \quad (104)$$

This rule maintains the rate of nominal depreciation against the dollar equal to a predetermined path δ_t^* that eventually converges to a constant ($\bar{\delta}^*$). Hence, the following equation must be included in the system:

$$\delta_t = \delta_t^* \quad \forall t. \quad (105)$$

6.3 A multilaterally fixed exchange rate or rate or crawl (MFIIX)

Consider now a multilateral pegging of the rate of crawl of the exchange rate to the multilateral trade-weighted basket of currencies. In this case, the Central Bank intervenes in the foreign exchange market so as to ensure that the nominal exchange rate with the basket of currencies (S_t/ρ_t) grows according to a predetermined path for the rate of crawl $\{\delta_t^*\}$ that eventually converges to a constant. For this, the Central Bank's policy rule can be formalized as:

$$R_t = \lim_{k_1 \rightarrow \infty} R_{t-1} \left(\frac{S_t/S_{t-1}}{\rho_t/\rho_{t-1}} \right)^{-k_1} \delta_t^*.$$

Assuming that the operational target for foreign exchange market intervention is the peso/dollar exchange rate, the following is the exchange rate policy equation that must be included in the system:

$$\delta_t = \left(\frac{\rho_t}{\rho_{t-1}} \right) \delta_t^* \quad \forall t. \quad (106)$$

6.4 Inflation targeting regimes

Under Inflation Targeting there are various possibilities for monetary policy feedback rules that can define the Central Bank's operational target (the nominal domestic currency interest rate). A fairly general one is one where the Central Bank responds to expected deviations of the gross (headline) inflation rate from a target path $\{\pi_t^*\}$, to deviations of the gross wage inflation from a target path, and to deviations of the output level from a target path. We also introduce a preference for slow changes in the nominal exchange rate:

$$1 + i_t = \left(\frac{\pi_t^*}{\beta}\right)^{1-h_0} (1 + i_{t-1})^{h_0} E_t \left(\frac{\pi_{t+1}}{\pi_{t+1}^*}\right)^{h_1} \left(\frac{\pi_{W,t}}{\pi_{W,t}^*}\right)^{h_2} \left(\frac{y_t}{y_t^*}\right)^{h_3} \quad (107)$$

$$h_0 \in [0, 1], h_1 > 1, h_2 \geq 0, h_3 \geq 0.$$

We assume that this rule has the so-called "Taylor property" ($h_1 > 1$) whereby the Central Bank responds to excess goods inflation by increasing the expected real interest rate (and not merely the nominal interest rate). We have assumed that the interest rate smoothing coefficient h_0 is not greater than one, but note that we could have greater generality by allowing for "superinertial" policy rules for the nominal interest rate ($h_0 > 1$). If $h_2 > 0$, the target paths for the rates of inflation of goods and wages must converge to the same level $\bar{\pi}^*$. We also assume that the target path for output $\{y_t^*\}$ converges to the long run average (or non-stochastic steady state) output \bar{y} . Note that this target path could be the "natural" rate of output, as is often assumed. But we prefer to be more general in view of the arguments invoked in the introduction. Also note that, in contrast to Woodford (2003) in his New Keynesian model, we are constructing the interest rate rule so as to ensure a zero steady state output gap no matter what the steady state target for inflation is.

A variant of the interest rate feedback rule makes the Central Bank respond to "core" inflation, which may be defined as the rate of inflation of sticky goods' prices, instead of "headline" inflation. In that case π_t must be replaced by $\pi_{N,t}$ in (107). Another variant has a purely backward looking reaction function, replacing the expected deviation of inflation from target by the current deviation, as is the baseline case in Woodford (2003).

6.4.1 Inflation targeting under a pure float (IT-PF)

When there is a pure float the Central Bank abstains from intervening in the foreign exchange market. Hence, its stock of international reserves does not change:

$$R_t = R_0 \quad \forall t. \quad (108)$$

6.4.2 Inflation targeting under a managed float (IT-MF)

Alternatively, under a managed float the Central Bank actively intervenes in the foreign exchange market. We assume that the operational target is the level of international reserves, that there can be a preference for smoothing the variations in the level of international reserves, and that there is a long run target (γ^*) for

the ratio between international reserves and government foreign debt (which here matures each period):

$$R_t = (\gamma^* \bar{D})^{1-k_0} (R_{t-1})^{k_0} \left(\frac{\delta_t}{\bar{\pi}^*} \right)^{-k_1} \quad (109)$$

$$k_0 \in [0, 1), k_1 > 0.$$

Note that under this policy feedback rule the Central Bank does not aim at any specific level of the nominal exchange rate. However, it does have a policy of "leaning against the wind" by purchasing reserves whenever the peso appreciation against the dollar ($\delta_t < \bar{\pi}^*$) is strong enough. The nominal anchor is clearly the target inflation rate, as when there is a pure float.

7. The non-stochastic steady state

We now define the non-stochastic steady states around which we make log-linear approximations to the different dynamic systems that correspond to the alternative monetary policy regimes. Replacing the variables by their non-stochastic steady state values we obtain the following 20 common equations plus two additional equations that specify the alternative regimes:

$$\bar{G}^P F_N'(F_N^{-1}(\bar{y}_N/\bar{z}^N)) = \mu_F \bar{w} \quad (110)$$

$$\bar{G}^W \bar{\lambda} \bar{w} \bar{z}^H = \mu_H \bar{e}^\theta v'(\bar{L}) \quad (111)$$

$$\bar{c}^{\xi+(1-\xi)\sigma} \bar{\lambda} \varphi(\bar{m}/\bar{c}) = \kappa_1(1 - \beta\xi). \quad (112)$$

$$1 + \tau'(\bar{m}/\bar{c}) = \beta/\bar{\pi} \quad (113)$$

$$1 = \beta(1 + \bar{i})/\bar{\pi} \quad (114)$$

$$1 + \bar{i} = (1 + \bar{i}^*)\bar{\delta} \quad (115)$$

$$1 + \bar{i}^* = (1 + \bar{r}^*)[1 + \bar{\zeta} + p(\bar{D} - \bar{R})] \quad (116)$$

$$\bar{y}_N = [(1 + \bar{g}^*)\tau(\bar{m}/\bar{c})(1 - \theta)\bar{e}^\theta \bar{c}] \frac{1}{1 - x(\log \bar{\pi}_N)} \quad (117)$$

$$\bar{L} \equiv \left[(F_X')^{-1} \left(\frac{\bar{w}}{\bar{e}\bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) + F_N^{-1} \left(\frac{\bar{y}_N}{\bar{z}^N} \right) \right] \frac{1}{1 - x(\log \bar{\pi}_W)} \quad (118)$$

$$\bar{m} + \bar{b} = \bar{s}\bar{R} \quad (119)$$

$$\bar{T}\bar{B} = \bar{i}^*\bar{D} - \bar{r}^*\bar{R} \quad (120)$$

$$\bar{\delta} = \bar{\pi}_N \quad (121)$$

$$\bar{\pi} = \bar{\pi}_N \quad (122)$$

$$\bar{\pi}_W = \bar{\pi}_N \quad (123)$$

$$\bar{G}^P \equiv 1 - x(\log \bar{\pi}_N) + \frac{1}{\nu - 1} \left\{ \left(1 - \frac{1}{1 + \bar{i}} \bar{\pi}_N \right) x'(\log \bar{\pi}_N) \right\} \quad (124)$$

$$\bar{G}^W \equiv 1 - x(\log \bar{\pi}_W) + \frac{1}{\psi - 1} \{ (1 - \beta) x'(\log \bar{\pi}_W) \}. \quad (125)$$

$$\bar{s} \equiv \frac{\bar{\rho}}{\bar{\phi}} \bar{e}^{1-\theta} \quad (126)$$

$$\bar{y}_X = \bar{z}^X F_X \left((F_X')^{-1} \left(\frac{\bar{w}}{\bar{e} \bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) \right) \quad (127)$$

$$\bar{y} = \bar{e}^{1-\theta} \left(1 - \frac{\bar{\epsilon}}{\bar{\phi}} \right) \bar{y}_X + \frac{\bar{y}_N}{\bar{e}^\theta} [1 - x(\log \bar{\pi}_N)] \quad (128)$$

$$\overline{TB} = \left(\frac{\bar{\phi} - \bar{\epsilon}}{\bar{\rho}} \right) \bar{y}_X - \frac{1}{\bar{s}} (1 + \bar{g}^*) \tau \left(\frac{\bar{m}}{\bar{c}} \right) \theta \bar{c} \quad (129)$$

UFIX and MFIX:

$$\bar{b} = b_0 \quad (130)$$

$$\bar{\delta} = \bar{\delta}^*. \quad (131)$$

IT-PF:

$$(1 + \bar{i})^{1-h_0} = \left(\frac{\bar{\pi}^*}{\beta} \right)^{1-h_0} \left(\frac{\bar{\pi}}{\bar{\pi}^*} \right)^{h_1} \left(\frac{\bar{\pi}_W}{\bar{\pi}^*} \right)^{h_2} \left(\frac{\bar{y}}{\bar{y}^*} \right)^{h_3} \quad (132)$$

$$\bar{R} = R_0. \quad (133)$$

IT-MF:

$$(1 + \bar{i})^{1-h_0} = \left(\frac{\bar{\pi}^*}{\beta} \right)^{1-h_0} \left(\frac{\bar{\pi}}{\bar{\pi}^*} \right)^{h_1} \left(\frac{\bar{\pi}_W}{\bar{\pi}^*} \right)^{h_2} \left(\frac{\bar{y}}{\bar{y}^*} \right)^{h_3} \quad (134)$$

$$\bar{R}^{1-k_0} = (\gamma^* \bar{D})^{1-k_0} \left(\frac{\bar{\delta}}{\bar{\pi}^*} \right)^{-k_1}. \quad (135)$$

7.1 Reduced steady state systems

The preceding steady state systems can easily be reduced to 6 equations. We will later reduce them even further to the traditional Internal and External balance equations.

7.1.1 FIX regimes

Starting with the FIX regimes, (131) and (121) to (123) imply

$$\bar{\pi}_N = \bar{\pi}_W = \bar{\pi} = \bar{\delta} = \bar{\delta}^*. \quad (136)$$

Hence, (114) and (115) imply

$$1 + \bar{i} = \bar{\delta}^* / \beta, \quad 1 + \bar{i}^* = 1 / \beta. \quad (137)$$

Then (113) and (114) imply:

$$\bar{m} = \ell \left(\bar{\delta}^* / \beta \right) \bar{c}, \quad (138)$$

where the liquidity preference function $\ell(\cdot)$ was defined in (58). Hence, we use this expression to eliminate \bar{m} and, in particular, we replace $\tau(\bar{m}/\bar{c})$ by $\bar{\tau}(\bar{\delta}^*/\beta)$ and $\varphi(\bar{m}/\bar{c})$ by $\bar{\varphi}(\bar{\delta}^*/\beta)$. Therefore, (119), (126), (130) and (138) determine \bar{R} as:

$$\bar{R} = \frac{\bar{\phi}}{\bar{\rho} \bar{e}^{1-\theta}} \left[\ell \left(\bar{\delta}^* / \beta \right) \bar{c} + b_0 \right],$$

and (116) determines the steady state value of the government debt in terms of \bar{e} , \bar{c} , and exogenous steady state variables and parameters:

$$\bar{D} = p^{-1} \left(\frac{1/\beta}{1 + \bar{r}^*} - 1 - \bar{\zeta} \right) + \frac{\bar{\phi}}{\bar{\rho} e^{1-\theta}} \left[\ell \left(\bar{\delta}^* / \beta \right) \bar{c} + b_0 \right]$$

Also, using (13) in (124) and (125) yields $\bar{G}^P = \bar{G}^W = 1$. We are left with the following 6 equation system that presumably determines $\bar{y}_N, \bar{w}, \bar{\lambda}, \bar{c}, \bar{e}, \bar{L}$:

$$F_N'(F_N^{-1}(\bar{y}_N/\bar{z}^N)) = \mu_F \bar{w} \quad (139)$$

$$\bar{\lambda} \bar{w} \bar{z}^H = \mu_H e^\theta v'(\bar{L}) \quad (140)$$

$$\bar{c}^{\xi+(1-\xi)\sigma} \bar{\lambda} \bar{\varphi}(\bar{\delta}^*/\beta) = \kappa_1(1 - \beta\xi). \quad (141)$$

$$\bar{y}_N = (1 + \bar{g}^*) \bar{\tau}(\bar{\delta}^*/\beta)(1 - \theta) \bar{e}^\theta \bar{c} \quad (142)$$

$$\bar{L} = (F_X')^{-1} \left(\frac{\bar{w}}{\bar{e} \bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) + F_N^{-1} \left(\frac{\bar{y}_N}{\bar{z}^N} \right) \quad (143)$$

$$\begin{aligned} \overline{TB}(\bar{e}, \bar{w}, \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{\delta}^*) = \\ \left(\frac{1}{\beta} - 1 \right) p^{-1} \left(\frac{1/\beta}{1 + \bar{r}^*} - 1 - \bar{\zeta} \right) + \left[\frac{1}{\beta} - 1 - \bar{r}^* \right] \frac{\bar{\phi}}{\bar{\rho} e^{1-\theta}} \left[\ell \left(\bar{\delta}^* / \beta \right) \bar{c} + b_0 \right] \end{aligned} \quad (144)$$

where we have defined the trade balance function as:

$$\begin{aligned} \overline{TB}(\bar{e}, \bar{w}, \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{\delta}^*) = \\ = \frac{\bar{\phi}}{\bar{\rho}} \left[\left(1 - \bar{\epsilon}/\bar{\phi} \right) \bar{z}^X F_X \left((F_X')^{-1} \left(\frac{\bar{w}}{\bar{e} \bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) \right) - (1 + \bar{g}^*) \bar{\tau} \left(\frac{\bar{\delta}^*}{\beta} \right) \theta \frac{\bar{c}}{\bar{e}^{1-\theta}} \right], \\ \overline{TB}_e > 0, \overline{TB}_w < 0, \overline{TB}_c < 0, \overline{TB}_\rho < 0, \overline{TB}_\phi > 0, \\ \overline{TB}_\epsilon < 0, \overline{TB}_{g^*} < 0, \overline{TB}_{z^X} > 0, \overline{TB}_{\delta^*} < 0. \end{aligned} \quad (145)$$

The first two equations are the flexible price monopolistic setting of non-tradable prices and wages, respectively. The following three equations are the consumption equation and the market clearing conditions (or resource constraints) for non-tradable goods and labor, respectively. Finally, the last equation is the balance of payments, which shows that the trade surplus must be equal to the risk adjusted interest payments on the dollar government debt in the hands of non-residents, net of the (risk-free) interest earned on Central Bank international reserves. The signs of \overline{TB}_ρ and \overline{TB}_ϕ assume that the non-stochastic steady state is positive (i.e., there is a positive non-stochastic steady state foreign debt level).

7.1.2 IT-PF regime

In this case, first note that we assumed that the Central Bank's target path for output in the interest rate feedback rule converges to the non-stochastic steady state output (if it ever departs) $\bar{y}^* = \bar{y}$, and that (122) and (123) imply $\bar{\pi}_W = \bar{\pi}$. Then, inserting these equalities and (114) in (132) gives:

$$\left(\frac{\bar{\pi}}{\bar{\pi}^*} \right)^{h_1+h_2+h_0-1} = 1, \quad (146)$$

which implies $\bar{\pi} = \bar{\pi}^*$ since $h_1 + h_2 \geq h_1 > 1 \geq 1 - h_0$. Therefore, we obtain

$$\bar{\pi}_N = \bar{\pi}_W = \bar{\pi} = \bar{\delta} = \bar{\pi}^*. \quad (147)$$

and (137) and (138) except that now the steady state gross nominal interest rate is $\bar{\pi}^*/\beta$ (instead of $\bar{\delta}^*/\beta$). The first 5 equations in the 6 equation system are the same as in the fixed exchange rate system, and only equation (144) changes to:

$$\begin{aligned} \overline{TB}(\bar{e}, \bar{w}, \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{\pi}^*) = \\ = \left(\frac{1}{\beta} - 1\right) p^{-1} \left(\frac{1/\beta}{1 + \bar{r}^*} - 1 - \bar{\zeta}\right) + \left(\frac{1}{\beta} - 1 - \bar{r}^*\right) R_0, \end{aligned} \quad (148)$$

Note that in (145) we must substitute $\bar{\pi}^*$ for $\bar{\delta}^*$.

7.1.3 IT-MF regime

Since we have the same interest rate feedback rule as under a pure float, we again obtain (146) and (147). Hence, (135) yields

$$\bar{R} = \gamma^* \bar{D},$$

confirming that the steady state target for the international reserves to foreign debt ratio is obtained in the long run if shocks are at their average values. Again, the first five equations are the same as for the two previous regimes and the only change is in the balance of payments equation, which becomes:

$$\overline{TB}(\bar{e}, \bar{w}, \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{\pi}^*) = \left(\frac{1/\beta - 1 - \bar{r}^* \gamma^*}{1 - \gamma^*}\right) p^{-1} \left(\frac{1/\beta}{1 + \bar{r}^*} - 1 - \bar{\zeta}\right). \quad (149)$$

7.2 Steady state Internal and External Balance

We now proceed to further eliminate variables so as to end up with a two equation system that determines \bar{e} and \bar{c} . Consider the IT-PF regime. First, note that (142) expresses non-tradable output in terms of those two endogenous variables:

$$\bar{y}_N = \beta_w \bar{e}^\theta \bar{c}, \quad \beta_w \equiv (1 + \bar{g}^*) \bar{\tau}(\bar{\pi}^*/\beta)(1 - \theta).$$

Also note that a higher steady state inflation rate implies a higher steady state nominal rate and a higher opportunity cost from holding money. Hence, households economize on money holdings, which implies higher transaction costs. This implies a higher use of real resources, which have to be produced. Therefore, a higher steady state inflation implies a higher steady state output, which is solely due to non utility generating consumption of real resources in transaction costs.

Using this expression to eliminate \bar{y}_N from (139) yields:

$$\begin{aligned} \bar{w} = (1/\mu_F) F_N'(F_N^{-1}(\beta_w \bar{e}^\theta \bar{c}/\bar{z}^N)) \equiv \bar{w}(\bar{e}^\theta \bar{c}; \bar{g}^*, \bar{z}^N, \mu_F, \bar{\pi}^*), \\ \bar{w}' < 0, \quad \bar{w}_{g^*} < 0, \quad \bar{w}_{z^N} > 0, \quad \bar{w}_{\mu_F} < 0, \quad \bar{w}_{\bar{\pi}^*} < 0. \end{aligned} \quad (150)$$

Inserting the last two expressions in (143) gives employment in terms of \bar{e} and \bar{c} :

$$\begin{aligned} \bar{L} = (F_X')^{-1} \left(\frac{\bar{w}(\bar{e}^\theta \bar{c}; \bar{g}^*, \bar{z}^N, \mu_F, \bar{\pi}^*)}{\bar{e} \bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) + F_N^{-1} \left(\frac{(1 + \bar{g}^*) \bar{\tau}(\bar{\pi}^*/\beta)(1 - \theta) \bar{e}^\theta \bar{c}}{\bar{z}^N} \right) \\ \equiv \bar{L}(\bar{e}, \bar{c}; \bar{g}^*, \bar{z}^N, \bar{z}^X, \bar{\phi}, \bar{\epsilon}, \mu_F, \bar{\pi}^*) \\ \bar{L}_e > 0, \quad \bar{L}_c > 0, \quad \bar{L}_{g^*} > 0, \quad \bar{L}_{z^N} < 0, \quad \bar{L}_{z^X} > 0, \\ \bar{L}_\phi > 0, \quad \bar{L}_\epsilon < 0, \quad \bar{L}_{\mu_F} > 0, \quad \bar{L}_{\bar{\pi}^*} > 0. \end{aligned}$$

Note that all the partial derivative have unambiguous signs. Now use (141) to eliminate $\bar{\lambda}$ from (140) and use the functions for \bar{w} and \bar{L} to obtain the Internal Balance (IB) equation :

$$v'(\bar{L}(\bar{e}, \bar{c}; \bar{g}^*, \bar{z}^N, \bar{z}^X, \bar{\phi}, \bar{\epsilon}, \mu_F, \bar{\pi}^*)) \bar{e}^\theta \bar{c}^{\xi+(1-\xi)\sigma} = \beta_\kappa \bar{z}^H \bar{w}(\bar{e}^\theta \bar{c}; \bar{g}^*, \bar{z}^N, \mu_F, \bar{\pi}^*) / \mu_H$$

$$\beta_\kappa \equiv \frac{\kappa_1(1 - \beta\xi)}{\bar{\varphi}(\bar{\pi}^*/\beta)}.$$

It is convenient to rearrange this equation as the equality of labor demand and labor supply:

$$\bar{L}(\bar{e}, \bar{c}; \bar{g}^*, \bar{z}^N, \bar{z}^X, \bar{\phi}, \bar{\epsilon}, \mu_F, \bar{\pi}^*) = (v')^{-1} \left(\frac{\beta_\kappa \bar{z}^H \bar{w}(\bar{e}^\theta \bar{c}; \bar{g}^*, \bar{z}^N, \mu_F, \bar{\pi}^*)}{\mu_H \bar{e}^\theta \bar{c}^{\xi+(1-\xi)\sigma}} \right) \quad (151)$$

$$\equiv \bar{L}^S(\bar{e}, \bar{c}; \bar{g}^*, \bar{z}^N, \bar{z}^H, \mu_F, \mu_H, \bar{\pi}^*)$$

$$\bar{L}_e^S < 0, \bar{L}_c^S < 0, \bar{L}_{g^*}^S < 0, \bar{L}_{z^N}^S > 0, \bar{L}_{z^H}^S > 0, \bar{L}_{\mu_F}^S < 0, \bar{L}_{\mu_H}^S < 0, \bar{L}_{\bar{\pi}^*}^S < 0$$

Given the signs of the partial derivatives, it is clear that the IB equation has a negative slope in the e-c plane, as depicted in Figure 1.

To obtain the External Balance equation, we first obtain the trade balance in terms of \bar{e} and \bar{c} by inserting (150) in (145) to obtain:

$$\bar{T}(\bar{e}, \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{z}^N, \mu_F, \bar{\pi}^*) \equiv$$

$$\bar{T}\bar{B}(\bar{e}, \bar{w}(\bar{e}^\theta \bar{c}; \bar{g}^*, \bar{z}^N, \mu_F, \bar{\pi}^*), \bar{c}; \bar{\rho}, \bar{\phi}, \bar{\epsilon}, \bar{g}^*, \bar{z}^X, \bar{\pi}^*)$$

$$\bar{T}_e > 0, \bar{T}_c < 0?, \bar{T}_\rho < 0, \bar{T}_\phi > 0, \bar{T}_\epsilon < 0,$$

$$\bar{T}_{g^*} < 0?, \bar{T}_{z^X} > 0, \bar{T}_{z^N} < 0, \bar{T}_{\mu_F} > 0, \bar{T}_{\bar{\pi}^*} < 0?$$

We have three partial derivatives with ambiguous sign. We shall assume that the direct effects of changes in \bar{c} , \bar{g}^* , and $\bar{\pi}^*$ through the demand for exportables predominate over the indirect effects through the product wage in the exportable sector. To see if this is a reasonable assumption, note that we can also express the trade balance as a function of $\beta_w \bar{e}^\theta \bar{c}$ and \bar{e} :

$$\tilde{T}(\beta_w \bar{e}^\theta \bar{c}, \bar{e}; \cdot) = \frac{\bar{\phi}}{\bar{\rho}} \left[\left(1 - \bar{\epsilon}/\bar{\phi} \right) \bar{z}^X F_X \left((F_X')^{-1} \left(\frac{\bar{w}(\beta_w \bar{e}^\theta \bar{c}/\bar{z}^N; \mu_F)}{\bar{e} \bar{z}^X} \frac{\bar{\phi}}{\bar{\phi} - \bar{\epsilon}} \right) \right) \right.$$

$$\left. - \frac{\theta}{1 - \theta} \frac{\beta_w \bar{e}^\theta \bar{c}}{\bar{e}} \right],$$

$$\tilde{T}_1 < 0?, \quad \tilde{T}_e > 0.$$

The sign of the first partial derivative is in general ambiguous because (given \bar{e} , an increase in $\beta_w \bar{e}^\theta \bar{c}$ increases the consumption of exportables but also increases the production of exportables through a reduction in the product wage in this sector. The net effect is given by the following partial derivative:

$$\tilde{T}_1 \equiv \frac{\partial \tilde{T}}{\partial (\beta_w \bar{e}^\theta \bar{c})} = \frac{\bar{\phi}}{\bar{\rho}} \left[\frac{F_X''/F_X'}{F_X''/F_X'} \frac{1}{\mu_F \bar{z}^N} - \frac{\theta}{1 - \theta} \frac{1}{\bar{e}} \right],$$

which is negative if and only if the following condition holds

$$\frac{F''_N/F'_N}{F''_X/F'_X} \frac{1}{\mu_F \bar{z}^N} < \frac{\theta}{1-\theta} \frac{1}{\bar{e}} = \frac{\bar{c}_X}{\bar{c}_N}. \quad (152)$$

To be specific, let the production functions be

$$F_K = \beta_K (L_K)^{\alpha_K}, \quad (0 < \alpha_K < 1) \quad K = X, N.$$

Then, using (47), we see that \tilde{T}_1 is negative if and only if the relative employment in the exportable sector is sufficiently small in comparison to the relative domestic consumption of exportables:

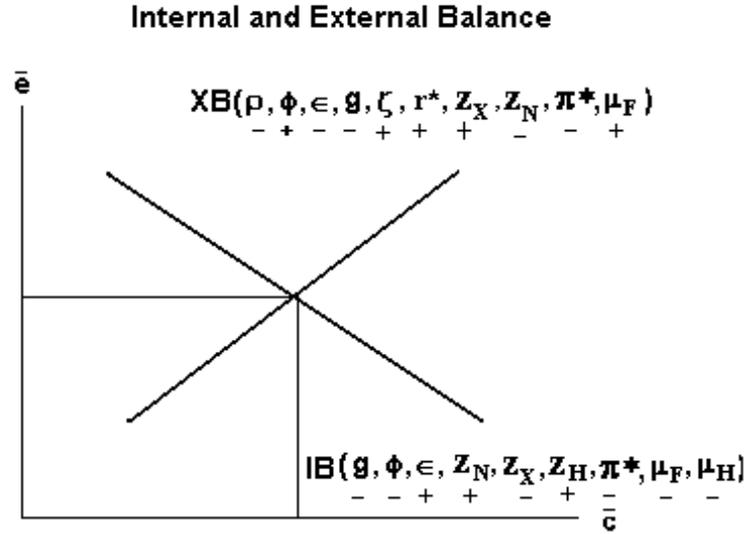
$$\frac{1 - \alpha_N \bar{L}_X}{1 - \alpha_X \bar{L}_N} \frac{1}{\mu_F \bar{z}^N} < \frac{\theta}{1-\theta} \frac{1}{\bar{e}} = \frac{\bar{c}_X}{\bar{c}_N}. \quad (153)$$

This seems an acceptable assumption for a country as Argentina, where goods account for around 1/2 of the CPI index (making the r.h.s. around 1) whereas employment in the goods producing sectors has always been much smaller than in the services producing sectors while the labor share in income is higher in the non tradable sector ($\alpha_N > \alpha_X$). Then assuming usual values for the markup μ_F and a normalization of units such that the steady state productivity parameter in the non tradable sector \bar{z}^N is in the vicinity of one, this inequality should hold for countries similar to Argentina. Taking into account that our long run is actually a medium run, where we take as given the relative amounts of capital in the two sectors, this historical fact can be assumed to be valid in the model's long run, i.e. the non-stochastic steady state.

Under the assumption that (152) holds, the External Balance line has a positive slope in the c-e plane, as depicted in Figure 1. We use the same diagram for all the regimes, but it is clear that the External Balance line is not necessarily the same for all the regimes. Furthermore, note that under the assumption that (152) holds, the non-stochastic steady state, if it exists, is necessarily globally unique.

We can now easily see the effects of changes in the mean values of exogenous variables on the long run equilibrium values of the MRER and private consumption. For example, a permanent dollar strengthening (a rise in $\bar{\rho}$) has the effect of shifting the XB line leftward (which is what the minus sign underneath this variable means in the graph). This makes the steady state MRER increase and private consumption of private goods decline. The reason, of course, is that the strengthening of the dollar makes the trade weighted trade surplus decline in dollar terms. To restore the dollar trade surplus, the MRER must increase, given c, or c must decline, given e. In fact, the shift in XB makes both e rise and c fall, the relative magnitude of these changes being given by slope of the Internal Balance line.

Other unambiguous effects are the following. An increase in the terms of trade ($\bar{\phi}$) makes e decline, because both the trade surplus and the net demand for labor increase, and both effects reduce the MRER needed for a given consumption. An opening of the economy based on higher import requirements for the exportable sector ($\bar{\epsilon}$) makes e increase through the increased demand for foreign exchange. An increase in the international dollar interest rate (\bar{r}^*) or in the exogenous component



of the risk premium ($\bar{\zeta}$) makes e fall and c increase. The reason is that the steady state government debt falls, and hence lower interest payments must be made. An increase in the provision of public goods (or, equivalently, an increase in public expenditures \bar{g}) makes private consumption of private goods fall. Note, however, that private welfare could either increase or fall, depending on the specifics of the subutility functions. An increase in productivity in the non tradable (exportable) sector increases (decreases) the MRER. An increase in the participation rate \bar{z}^H (or willingness to work) increases e and c because the higher labor income leads to greater consumption of exportable goods (as well as non-tradable goods), which lowers the trade surplus, requiring a greater steady state MRER.

We can also gauge the model's implications for changes in the long run inflation induced by monetary policy. An increase in steady state inflation increases the nominal interest rate. Hence, agents economize on their money holdings, which implies an increase in the use of real resources in transactions. Since this affects both exportable and non tradable goods, both lines shift to the left and consumption of private goods falls.

Figure 1 also shows the effects of structural reforms pertaining to the monopoly power of price and wage setters, as given by the elasticities of substitution of non-tradable goods and labor specializations, respectively. An increase in the monopoly power of non-tradable firms reduces the product wage there, as well as in the exportable sector. Labor demand increases and labor supply falls, while the trade surplus increases. The combination of both shifts makes the MRER fall. Finally, an increase in the monopoly power of households leads them to work less and thus both e and c fall.

8. Coexistence of backward and forward looking firms

When we have both backward and forward looking firms we must introduce new equations in the non linear systems and also modify some of the old equations. The modifications respond to the need to specify, in some of the equations, variables that pertain to forward or to backward looking firms. Specifically, we must replace equations (83), (90), (91), and (98) by:

$$G_t^P = \mu_F \frac{w_t^f}{F_N'(F_N^{-1}(y_{N,t}^f/z_t^F))} \quad (154)$$

$$y_{N,t} = [(1 + g_t^*)\bar{\tau}(1 + i_t)(1 - \theta)e_t^\theta c_t] \left[1 + \varsigma_F \frac{x(\log \pi_{N,t}^f)}{1 - x(\log \pi_{N,t}^f)} \right] \quad (155)$$

$$L_t \equiv \left[(F_X')^{-1} \left(\frac{w_t}{e_t z_t^X} \frac{\phi_t}{\phi_t - \epsilon_t} \right) + \varsigma_F F_N^{-1} \left(\frac{y_{N,t}^f}{z_t^N} \right) \right. \\ \left. + (1 - \varsigma_F) F_N^{-1} \left(\frac{y_{N,t}^b}{z_t^N} \right) \right] \frac{1}{1 - x(\log \pi_{W,t})} \quad (156)$$

$$G_t^P = 1 - x(\log \pi_{N,t}^f) + \\ + \frac{1}{\nu - 1} \left\{ x'(\log \pi_{N,t}^f) - E_t \left[\frac{1}{1 + i_t} \frac{y_{N,t+1}^f}{y_{N,t}^f} \pi_{N,t+1}^f x'(\log \pi_{N,t+1}^f) \right] \right\}. \quad (157)$$

And the new equations are the following:

$$w_t^f = w_t/p_{N,t}^f. \\ y_{N,t}^f = y_{N,t} (p_{N,t}^f)^{-\nu} \\ y_{N,t}^b = y_{N,t} (p_{N,t}^b)^{-\nu} \\ \pi_{N,t}^f = (p_{N,t}^f/p_{N,t-1}^f) \pi_{N,t} \\ \pi_{N,t}^b = (p_{N,t}^b/p_{N,t-1}^b) \pi_{N,t} \\ \pi_{N,t}^b = \pi_{N,t-1} + \alpha \bar{\pi}_N (p_{N,t-1} - 1). \\ p_{N,t} = p_{N,t}^f/p_{N,t}^b \\ 1 = \varsigma_F (p_{N,t}^f)^{1-\nu} + (1 - \varsigma_F) (p_{N,t}^b)^{1-\nu}.$$

These 8 new equations account for the following 8 new variables: w_t^f , $y_{N,t}^f$, $y_{N,t}^b$, $p_{N,t}^f$, $p_{N,t}^b$, $\pi_{N,t}^f$, $\pi_{N,t}^b$, $p_{N,t}$.

In the steady state, these new equations are:

$$\bar{w}^f = \bar{w}/\bar{p}_N^f. \quad (158)$$

$$\bar{y}_N^f = \bar{y}_N \left(\bar{p}_N^f \right)^{-\nu} \quad (159)$$

$$\bar{y}_N^b = \bar{y}_N \left(\bar{p}_N^b \right)^{-\nu} \quad (160)$$

$$\bar{\pi}_N^f = \bar{\pi}_N \quad (161)$$

$$\bar{\pi}_N^b = \bar{\pi}_N \quad (162)$$

$$\bar{\pi}_N^b = \bar{\pi}_N + \alpha \bar{\pi}_N (\bar{p}_N - 1). \quad (163)$$

$$\bar{p}_N = \bar{p}_N^f / \bar{p}_N^b \quad (164)$$

$$1 = \varsigma_F \left(\bar{p}_N^f \right)^{1-\nu} + (1 - \varsigma_F) \left(\bar{p}_N^b \right)^{1-\nu}. \quad (165)$$

(161) and (162) imply $\bar{\pi}_N^f = \bar{\pi}_N^b = \bar{\pi}_N$. Hence, (163) implies $\bar{p}_N = 1$, (164) and (165) imply $\bar{p}_N^b = \bar{p}_N^f = 1$, (158) implies $\bar{w}^f = \bar{w}$, and (159) and (160) imply $\bar{y}_N^f = \bar{y}_N^b = \bar{y}_N$. Therefore, the steady state of the modified equations (154)-(157) are the same as when all firms optimize. In short, the steady state values determined by the non linear system without "rule of thumb" firms remain valid, and the new equations determine the steady state values of the additional variables that arise upon the introduction of firm heterogeneity.

9. The log-linearized systems

We log-linearize the dynamic systems in a (small) neighborhood of the non-stochastic steady states, which we assume exist and are unique. The following are the 19 log-linearized equations that are shared by all the alternative regimes when there is firm heterogeneity, after which we list the two policy equations for each regime:

Non-policy equations:

Non-tradable inflation dynamics

$$\hat{\pi}_{N,t} = h_b \hat{\pi}_{N,t-1} + h_f E_t \hat{\pi}_{N,t+1} + h_{mc} \left\{ \hat{w}_t + a_y (\hat{y}_{N,t} - \hat{z}_t^N) \right\} + h_p \hat{p}_{N,t-1} \quad (166)$$

Non-tradable relative price dynamics

$$\hat{p}_{N,t} = k \hat{p}_{N,t-1} + (1/\varsigma_F) (\hat{\pi}_{N,t} - \hat{\pi}_{N,t-1}) \quad (167)$$

Wage inflation dynamics

$$\hat{\pi}_{W,t} = \beta E_t \hat{\pi}_{W,t+1} + \gamma_H \{ \theta \hat{c}_t - \hat{\lambda}_t - \hat{w}_t + a_L \hat{L}_t - \hat{z}_t^H \} \quad (168)$$

Aggregate consumption dynamics

$$\hat{c}_t = a_0 \hat{c}_{t-1} + a_1 E_t \hat{c}_{t+1} - a_2 \hat{\lambda}_t - a_3 \hat{i}_t, \quad (169)$$

Marginal utility of real income dynamics

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{i}_t - E_t \hat{\pi}_{t+1} \quad (170)$$

Uncovered interest parity

$$\widehat{i}_t = \widehat{i}_t^* + E_t \widehat{\delta}_{t+1} \quad (171)$$

Risk-adjusted dollar interest rate

$$\widehat{i}_t^* = \widehat{r}_t^* + \alpha_\zeta \widehat{\zeta}_t + (1 - \alpha_\zeta) \bar{\varepsilon}_p \left[\alpha_G \widehat{D}_t - (1 - \alpha_G) \widehat{R}_t \right] \quad (172)$$

Non-tradables market clearing

$$\widehat{y}_{N,t} = \widehat{c}_t + \theta \widehat{e}_t + a_I \widehat{i}_t + \widehat{g}_t^* \quad (173)$$

Labor market clearing

$$\widehat{L}_t = a_{LX} \left[\widehat{e}_t - \widehat{w}_t + \beta_\phi (\widehat{\phi}_t - \widehat{\varepsilon}_t) \right] + a_{LN} \widehat{y}_{N,t} + a_{LX} \widehat{z}_t^X - a_{LN} \widehat{z}_t^N \quad (174)$$

Money market clearing

$$\widehat{m}_t = \widehat{c}_t - \bar{\varepsilon}_\ell \widehat{i}_t \quad (175)$$

Central Bank balance sheet

$$\widehat{R}_t = \alpha_m \widehat{m}_t + (1 - \alpha_m) \widehat{b}_t - \widehat{s}_t \quad (176)$$

Balance of payments

$$\begin{aligned} -\widehat{TB}_t = \alpha_D \left\{ \widehat{D}_t - (\bar{\pi}^*/\beta) \widehat{D}_{t-1} - (\bar{\pi}^*/\beta) \widehat{i}_{t-1}^* \right\} - \\ - \alpha_R \left\{ \widehat{R}_t - (1 + \bar{r}^*) \widehat{R}_{t-1} - (1 + \bar{r}^*) \widehat{r}_{t-1}^* \right\} \end{aligned} \quad (177)$$

Identities

$$\widehat{\delta}_t = \widehat{\pi}_{N,t} + \widehat{e}_t - \widehat{e}_{t-1} + \widehat{\rho}_t - \widehat{\rho}_{t-1} - \widehat{\phi}_t + \widehat{\phi}_{t-1} \quad (178)$$

$$\widehat{\pi}_t = \theta (\widehat{e}_t - \widehat{e}_{t-1}) + \widehat{\pi}_{N,t} \quad (179)$$

$$\widehat{w}_t = \widehat{w}_{t-1} + \widehat{\pi}_{W,t} - \widehat{\pi}_{N,t} \quad (180)$$

$$\widehat{s}_t = (1 - \theta) \widehat{e}_t + \widehat{\rho}_t - \widehat{\phi}_t \quad (181)$$

Exportable output

$$\widehat{y}_{X,t} = \varepsilon_{FF} \left[\widehat{e}_t - \widehat{w}_t + \beta_\phi (\widehat{\phi}_t - \widehat{\varepsilon}_t) \right] + (1 + \varepsilon_{FF}) \widehat{z}_t^X \quad (182)$$

Aggregate output

$$\widehat{y}_t = (1 - \beta_y) \widehat{y}_{N,t} + \beta_y \left[\widehat{y}_{X,t} + (\beta_\phi - 1) (\widehat{\phi}_t - \widehat{\varepsilon}_t) \right] + (\beta_y - \theta) \widehat{e}_t \quad (183)$$

Trade balance

$$\begin{aligned} \widehat{TB}_t = \widehat{\phi}_t - \widehat{\rho}_t + \beta_X \left[\widehat{y}_{X,t} + (\beta_\phi - 1) (\widehat{\phi}_t - \widehat{\varepsilon}_t) \right] - \\ - (\beta_X - 1) \left[\widehat{c}_t + \widehat{g}_t^* - (1 - \theta) \widehat{e}_t + a_I \widehat{i}_t \right] \end{aligned} \quad (184)$$

Policy equations:

UFIK:

$$\begin{aligned}\widehat{\delta}_t &= 0 \\ \widehat{b}_t &= 0,\end{aligned}$$

MFIK:

$$\begin{aligned}\widehat{\delta}_t &= \widehat{\rho}_t - \widehat{\rho}_{t-1} \\ \widehat{b}_t &= 0,\end{aligned}$$

IT-PF:

$$\begin{aligned}\widehat{i}_t &= h_0 \widehat{i}_{t-1} + h_1 E_t (\widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^*) + h_2 (\widehat{\pi}_{W,t} - \widehat{\pi}_{W,t}^*) + h_3 (\widehat{y}_t - \widehat{y}_t^*) \\ \widehat{R}_t &= 0,\end{aligned}\tag{185}$$

IT-MF:

$$\begin{aligned}\widehat{i}_t &= h_0 \widehat{i}_{t-1} + h_1 E_t (\widehat{\pi}_{t+1} - \widehat{\pi}_{t+1}^*) + h_2 (\widehat{\pi}_{W,t} - \widehat{\pi}_{W,t}^*) + h_3 (\widehat{y}_t - \widehat{y}_t^*) \\ \widehat{R}_t &= k_0 \widehat{R}_{t-1} - k_1 \widehat{\delta}_t.\end{aligned}\tag{186}$$

Except for the four dynamic equations (178)-(181), which are the log-linear versions of the identities (94)-(97), and the policy equations, the rest of the equations have already been encountered. Equation (166) is our "New Keynesian Phillips Curve" for non tradable goods and (167) gives the dynamics for the relative price between goods produced by forward versus backward looking firms. Equation (168) is the "New Keynesian Phillips Curve" for wages. (169) and (170) give the dynamics for consumption and the marginal utility of real income, respectively. (171) is the UIP equation and (172) is the (risk-adjusted) dollar interest rate on government external debt. (173) to (175) are the market clearing equations for non tradable goods, labor, and money, respectively. (176) is the Central Bank full backing condition (or balance sheet "identity"). (177) is the balance of payments equation, (178)-(181) are derived from the definitions of δ , e , w , and s . (182)-(184) are the expressions for exportable output, total output, and the trade balance, respectively. Finally, the subsequent 4 pairs of equations are the policy rules that correspond to the 4 alternative regimes.

The 21 equations in each regime determine the paths of the following 21 variables: $\widehat{\pi}_{N,t}$, $\widehat{p}_{N,t}$, $\widehat{\pi}_{W,t}$, \widehat{c}_t , $\widehat{\lambda}_t$, \widehat{i}_t^* , $\widehat{y}_{N,t}$, \widehat{L}_t , \widehat{m}_t , \widehat{b}_t , \widehat{D}_t , \widehat{e}_t , $\widehat{\pi}_t$, \widehat{w}_t , \widehat{s}_t , $\widehat{y}_{X,t}$, \widehat{y}_t , \widehat{TB}_t , $\widehat{\delta}_t$, \widehat{i}_t , \widehat{R}_t , given the paths of the exogenous stochastic variables $\widehat{\rho}_t$, $\widehat{\phi}_t$, $\widehat{\epsilon}_t$, \widehat{r}_t^* , $\widehat{\zeta}_t$, \widehat{z}_t^N , \widehat{z}_t^X , \widehat{z}_t^H , \widehat{g}_t . We may assume that the exogenous forcing variables follow a first order VAR process:

$$z_t = Qz_{t-1} + \varkappa_t, \quad \varkappa_t \sim iid(0, \Sigma_\varkappa),$$

where Q is a 7 by 7 matrix with all its roots inside the unit circle. Hence, the systems may be expressed in matrix form suitable for computing numerical solution(s) after calibrating the parameters (see Blanchard and Kahn (1980), Binder

and Pesaran (1995), Uhlig (1997) and Sims (2000)). In Uhlig's format, for example, we have:

$$\begin{aligned} 0 &= Ax_t + Bx_{t-1} + Cy_t + Dz_t \\ 0 &= E_t \{Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t\} \end{aligned}$$

where x_t is the vector of predetermined endogenous variables, y_t is the vector of non-predetermined (or "jump") endogenous variables, and z_t is the vector of exogenous variables:

$$z_t = (\widehat{g}_t, \widehat{r}_t^*, \widehat{\zeta}_t, \widehat{\phi}_t, \widehat{\rho}_t, \widehat{\epsilon}_t, \widehat{z}_t^N, \widehat{z}_t^X, \widehat{z}_t^H, \widehat{\pi}_t^*, \widehat{\pi}_{W,t}^*, \widehat{y}_t^*)^\top.$$

The assignment of the endogenous variables to the two vectors x_t and y_t (and hence, the matrices involved), and even what variables are to be included as exogenous, is dependant on the monetary policy regime. For example, under the FIX regimes we can eliminate the variables \widehat{b}_t , $\widehat{\delta}_t$, \widehat{i}_t , altogether (the latter because it is always equal to \widehat{i}_t^*). Furthermore, for these regimes the equation for \widehat{y}_t is decomposable from the rest, and hence may be eliminated from the core system. In the IT-PF regime we can eliminate \widehat{R}_t and note that the only equations with the variables \widehat{m}_t and \widehat{b}_t ((175) and (176)) are decomposable from the rest. This is no longer true under IT-MF, however, where the international reserves feedback rule generates interrelations with the monetary/Central Bank part of the system.

10. More on the Inflation Targeting with Managed Float regime

To understand the functioning of the model under inflation targeting with a managed float it is instructive to see what happens if (in (172)) we omit the endogenous component of the risk premium on dollar denominated government bonds, leaving only the exogenous component. In that case, \widehat{i}_t^* becomes exogenous. The nominal peso interest rate is affected by the UIP equation (171) and the interest rate rule (185). However, the Reserves Rule (186) has no effect whatsoever on $\widehat{\delta}_t$, $\widehat{\epsilon}_t$, or the inflation rates, because the equations (175), (176), (177), (181), (184), and (186), become decomposable from the rest. Hence, the Reserves Rule only affects the composition of the Central Bank balance sheet. For example, given a nominal appreciation ($\widehat{\delta}_t < 0$), an increase in the "leaning against the wind" coefficient k_1 makes \widehat{R}_t increase more over $k_0 \widehat{R}_{t-1}$. Since in (176) \widehat{m}_t and \widehat{s}_t are determined by the indecomposable part of the system it is \widehat{b}_t that increases when \widehat{R}_t increases (and increases more when there is an exogenous increase in k_1). Hence, the sole effect of the appreciation is to make the Central Bank purchase reserves and sterilize the monetary effect by issuing peso bonds, with no effective "leaning against the wind", i.e. ameliorating of the currency appreciation. This shows that the effectiveness of the managed float policy depends crucially on the endogenous component of the risk premium.

When the endogenous risk premium is in place, and there has been an appreciation, the increase in \widehat{R}_t reduces the risk premium and hence the dollar interest rate \widehat{i}_t^* . Given \widehat{i}_t^* (by the Interest Rule), the UIP equation generates an expected nominal depreciation. Furthermore, a stronger increase in \widehat{R}_t due to a rise in k_1 , implies

a lower \widehat{i}_t^* and hence a higher $E_t \widehat{\delta}_{t+1}$, given \widehat{i}_t . But the process is more complex, because the reduction in $\widehat{\delta}_t$ (from 0 to a negative value) produces an immediate fall in \widehat{e}_t (through (178)) and hence on $\widehat{\pi}_{W,t}$ (through (168)), $\widehat{\pi}_t$ (through (179)), $\widehat{y}_{X,t}$ (through (182)) and \widehat{y}_t (through (183)), and consequently on \widehat{i}_t through the effects of $\widehat{\pi}_{W,t}$, $E_t \widehat{\pi}_{t+1}$ and \widehat{y}_t on the Interest Rule. Hence, what actually happens to expected depreciation depends on the combined effects on \widehat{i}_t^* and \widehat{i}_t .

In our model for IT-MF, a Central Bank purchase of foreign exchange has the effect of reducing the risk premium on the government's foreign currency debt and hence the domestic dollar and peso interest rates. This is contrary to most accounts of the effects of Central Bank sterilized purchases of foreign exchange, where the domestic interest rate increases when there is imperfect substitution between the assets that households purchase. We have mentioned that our simple log-linear framework cannot include an adequate portfolio theory. For this, we would (at least) need a second order approximation to the model equations to obtain second moments (variances and covariances) for asset returns, and hence risk (see Obstfeld and Rogoff (1996)). Instead of attempting this, in the above framework we have chosen to keep the analysis simple and take advantage of the SOE assumption to introduce risk considerations through the (not modeled) portfolio decisions of international investors. A different, and admittedly ad hoc, way of introducing risk involves domestic households' assessment of the riskiness of Central Bank peso bonds and yields a story that is closer to most accounts of sterilized intervention. We elaborate on this in the rest of this section.

Assume that households believe that there is risk in investing in Central Bank bonds and that their perception of this risk is increasing in the amount of bonds outstanding. When the Central Bank issues additional bonds, it must make households willing to increase the risk in their portfolio by compensating them with a higher expected return, i.e. a risk premium. When households make their decision they discount the Central Bank's promised gross return by a risk factor $p_H(b_t)$ which is an increasing function of b_t .¹⁷ Hence, in the Lagrangian (50) the term that has the interest payments on the previous period's bond investment must be:

$$\frac{1 + i_{t-1+j} b_{t-1+j}}{p_H(b_{t-1+j}) \pi_{t+j}},$$

and the corresponding first order condition for b_t becomes (instead of (51c)):

$$1 = \beta(1 + i_t) \frac{1}{p_H(b_t)} \left[1 - \frac{b_t p'_H(b_t)}{p_H(b_t)} \right] E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{P_{t+1}}{P_t} \right). \quad (187)$$

For simplicity, let us assume that the risk premium is the following power function:¹⁸

$$p_H(b_t) = b_t^{\varepsilon_H},$$

¹⁷For analytical convenience we use the risk premium factor here, instead of the risk premium rate as previously.

¹⁸This simplicity has the cost of implying a concave risk premium function, which is contrary to intuition. A simple convex function is $p_H(b_t) = e^{ab_t}$, where a is a positive constant. However, in this case we would need to place an upper bound on b_t ($b_t < 1/a$) to exclude the elastic portion of the risk premium function.

where the constant ε_H , (which is strictly between 0 and 1), is the elasticity of $p_H(b_t)$. Hence, (187) becomes:¹⁹

$$1 = \beta(1 + i_t) \frac{1 - \varepsilon_H}{b_t^{\varepsilon_H}} E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \middle/ \pi_{t+1} \right).$$

This change affects a few of our system equations. First, although the liquidity preference function $\ell(\cdot)$ is formally the same as before and depends on the opportunity cost of holding money, this opportunity cost is now lower since the expected yield on peso bonds is lower. Hence, instead of (58), we have:

$$m_t = (-\tau')^{-1} \left(1 - \frac{1}{1 + i_t} \frac{b_t^{\varepsilon_H}}{1 - \varepsilon_H} \right) c_t \equiv \ell \left(\frac{(1 + i_t)(1 - \varepsilon_H)}{b_t^{\varepsilon_H}} \right) c_t.$$

Since the expected yield of Central Bank bonds is lower than when there is no risk premium, the opportunity cost of holding money is lower and, consequently, the stock of money is higher. Our auxiliary functions now also depend on this modified argument:

$$\bar{\tau} \left(\frac{(1 + i_t)(1 - \varepsilon_H)}{b_t^{\varepsilon_H}} \right), \quad \bar{\varphi} \left(\frac{(1 + i_t)(1 - \varepsilon_H)}{b_t^{\varepsilon_H}} \right).$$

Hence, all the equations that contain functions $\ell(\cdot)$, $\bar{\tau}(\cdot)$, or $\bar{\varphi}(\cdot)$ must be modified. The log linear versions of these modified equations are the following:

$$\widehat{c}_t = a_0 \widehat{c}_{t-1} + a_1 E_t \widehat{c}_{t+1} - a_2 \widehat{\lambda}_t - a_3 \left(\widehat{i}_t - \varepsilon_H \widehat{b}_t \right), \quad (188)$$

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{i}_t - \varepsilon_H \widehat{b}_t - E_t \widehat{\pi}_{t+1} \quad (189)$$

$$\widehat{i}_t = \widehat{i}_t^* + E_t \widehat{\delta}_{t+1} + \varepsilon_H \widehat{b}_t. \quad (190)$$

$$\widehat{y}_{N,t} = \widehat{c}_t + \theta \widehat{e}_t + a_I \left(\widehat{i}_t - \varepsilon_H \widehat{b}_t \right) + \widehat{g}_t^* \quad (191)$$

$$\widehat{m}_t = \widehat{c}_t - \bar{\varepsilon}_\ell \left(\widehat{i}_t - \varepsilon_H \widehat{b}_t \right) \quad (192)$$

Hence, if we insert (172) in (190) we see that there are now two risk premia that affect the peso interest rate:

$$\widehat{i}_t = \widehat{r}_t^* + \alpha_\zeta \widehat{\zeta}_t + (1 - \alpha_\zeta) \bar{\varepsilon}_p \left[\alpha_G \widehat{D}_t - (1 - \alpha_G) \widehat{R}_t \right] + E_t \widehat{\delta}_{t+1} + \varepsilon_H \widehat{b}_t.$$

A sterilized purchase of foreign exchange increases R_t and b_t , lowering the risk premium on the dollar interest rate $p(D_t - R_t)$ but increasing the risk premium on the peso interest rate $p_H(b_t)$. Hence, the effect on the peso interest rate depends on the specifics of these risk premia. This multiplicity of effects on different risk premia may explain, at least partly, the difficulties faced by those who attempt to empirically evaluate the effectiveness of sterilized intervention operations in the foreign exchange market. Different Central Banks engaging in sterilized intervention may face highly idiosyncratic situations as to the relative importance and effects of Central Bank international reserves and Central Bank domestic currency bonds (or Government bonds in the asset side of the Central Bank balance sheet) in the complex risk assessments that market participants make.

¹⁹We can also assume that the stock of bonds is greater than $(1 - \varepsilon_H)^{1/\varepsilon_H}$ in order to ensure that the gross risk premium is greater than one.

11. Some extensions

11.1 The "headline" Phillips equation

For empirical applications it may be of interest to see what the Phillips equation looks like if we choose to work with overall ("headline") consumer inflation, total output and the real wage (instead of non tradable inflation and output and the non tradable product wage). First, note that using (179) and the log-linear version of (6):

$$\widehat{w}_t^\circ = \widehat{w}_t - \theta \widehat{e}_t, \quad (193)$$

in (40) to eliminate $\widehat{\pi}_{N,t}$ and \widehat{w}_t gives:

$$\begin{aligned} \widehat{\pi}_t &= h_{b2}\widehat{\pi}_{t-2} + h_{b1}\widehat{\pi}_{t-1} + h_{f1}E_t\widehat{\pi}_{t+1} \\ &\quad - \theta \{h_{b2}(\widehat{e}_{t-2} - \widehat{e}_{t-3}) + h_{b1}(\widehat{e}_{t-1} - \widehat{e}_{t-2}) - (\widehat{e}_t - \widehat{e}_{t-1}) + h_{f1}(E_t\widehat{e}_{t+1} - \widehat{e}_t)\} \\ &\quad + h_{mc1} \{ \widehat{w}_t^\circ - k\widehat{w}_{t-1}^\circ + \theta[\widehat{e}_t - k\widehat{e}_{t-1}] + a_y [(\widehat{y}_{N,t} - \widehat{z}_t^N) - k(\widehat{y}_{N,t-1} - \widehat{z}_{t-1}^N)] \} \\ &\quad + h_\eta \eta_t. \end{aligned}$$

Finally, use (183) and (182), to substitute \widehat{y}_t for $\widehat{y}_{N,t}$ and rearrange to obtain the headline "hybrid" Phillips equation:

$$\begin{aligned} \widehat{\pi}_t &= h_{b2}\widehat{\pi}_{t-2} + h_{b1}\widehat{\pi}_{t-1} + h_{f1}E_t\widehat{\pi}_{t+1} - \theta \{h_{b2}(\widehat{e}_{t-2} - \widehat{e}_{t-3}) + h_{b1}(\widehat{e}_{t-1} - \widehat{e}_{t-2}) \\ &\quad - (\widehat{e}_t - \widehat{e}_{t-1}) + h_{f1}(E_t\widehat{e}_{t+1} - \widehat{e}_t)\} + h_{mc1} \{ k_e[\widehat{e}_t - k\widehat{e}_{t-1}] + k_y[\widehat{y}_t - k\widehat{y}_{t-1}] \\ &\quad + k_w[\widehat{w}_t^\circ - k\widehat{w}_{t-1}^\circ] - k_\phi \left[(\widehat{\phi}_t - \widehat{e}_t) - k(\widehat{\phi}_t - \widehat{e}_t) \right] - k_z(\widehat{z}_t^N - k\widehat{z}_{t-1}^N) \} \\ &\quad + h_\eta \eta_t, \end{aligned}$$

$$\begin{aligned} k_e &= \theta + a_y \left[1 - \frac{1 + \beta_y \varepsilon_{FF}}{1 - \beta_y} (1 - \theta) \right], \quad k_y = \frac{a_y}{1 - \beta_y} > 0, \\ k_w &= 1 + k_y \beta_y \varepsilon_{FF} > 0, \quad k_\phi = k_y [\beta_\phi \varepsilon_{FF} + \beta_\phi - 1] > 0, \\ k_z &= a_y \left[1 + \frac{\beta_y}{1 - \beta_y} (1 + \varepsilon_{FF}) \right] > 0. \end{aligned}$$

11.2 The IS and LM equations

We have chosen to work with the consumption dynamics equation and the non-tradable resource constraint separately. However, it is easy to collapse them into a typical IS equation. First, subtracting its lead from (169) and using (170) to eliminate λ_t yields:

$$\widehat{c}_t = b_0 \widehat{c}_{t-1} + b_1 E_t \widehat{c}_{t+1} - b_2 E_t \widehat{c}_{t+2} - b_3 (\widehat{i}_t - E_t \widehat{\pi}_{t+1}) - b_4 (\widehat{i}_t - E_t \widehat{i}_{t+1}). \quad (194)$$

$$b_0 = \frac{a_0}{1 + a_0}, b_1 = \frac{1 + a_1}{1 + a_0}, b_2 = \frac{a_1}{1 + a_0}, b_3 = \frac{a_2}{1 + a_0}, b_4 = \frac{a_3}{1 + a_0}.$$

Second, we eliminate $\widehat{y}_{N,t}$, $\widehat{y}_{X,t}$, and \widehat{w}_t from (183) as above, and use the resulting equation to obtain \widehat{c}_t in terms of \widehat{y}_t , \widehat{e}_t , \widehat{i}_t , \widehat{w}_t° , \widehat{g}_t , $\widehat{\phi}_t$, \widehat{e}_t , and \widehat{z}_t^F . Finally, lag once and lead twice on that equation to eliminate \widehat{c}_t and its lag and leads from (194).

This gives the following somewhat complicated "hybrid" IS equation:

$$\begin{aligned}\widehat{y}_t = & b_0\widehat{y}_{t-1} + b_1E_t\widehat{y}_{t+1} - b_2E_t\widehat{y}_{t+2} - b_3(1 - \beta_y)\left(\widehat{i}_t - E_t\widehat{\pi}_{t+1}\right) \\ & - b_4(1 - \beta_y)\left(\widehat{i}_t - E_t\widehat{i}_{t+1}\right) - \beta_{ye}P(\widehat{e}_t) + \beta_{yw}P(\widehat{w}_t^\circ) \\ & - (1 - \beta_y)\alpha_IP(\widehat{i}_t) - (1 - \beta_y)P(\widehat{g}_t) - \beta_{y\phi}P(\widehat{\phi}_t - \widehat{e}_t) - \beta_{yz}P(\widehat{z}_t^F)\end{aligned}$$

where we have defined:

$$\begin{aligned}\beta_{ye} &\equiv \beta_y(1 + \varepsilon_{FF})(1 - \theta) > 0, & \beta_{yw} &\equiv \beta_y\varepsilon_{FF} > 0, \\ \beta_{y\phi} &\equiv \beta_\phi\varepsilon_{FF} + \beta_\phi - 1 > 0, & \beta_{yz} &\equiv \beta_y(1 + \varepsilon_{FF}) > 0,\end{aligned}$$

$$P(x_t) \equiv b_0x_{t-1} - x_t + b_1E_tx_{t+1} - b_2E_tx_{t+2}.$$

Proceeding in a similar manner, we can use (69) to eliminate \widehat{c}_t from (61) and obtain an LM equation that shows how diverse the contemporaneous influences on the money stock are:

$$\widehat{m}_t = \ell_y\widehat{y}_t + \ell_w\widehat{w}_t^\circ - \ell_e\widehat{e}_t - (a_I + \bar{\varepsilon}_\ell)\widehat{i}_t - \widehat{g}_t - \ell_\phi(\widehat{\phi}_t - \widehat{e}_t) - \ell_z\widehat{z}_t^F$$

$$\ell_y \equiv \frac{1}{1 - \beta_y}, \ell_w \equiv \beta_{yw}\ell_y, \ell_e \equiv \beta_{ye}\ell_y - \theta, \ell_\phi \equiv \beta_{y\phi}\ell_y, \ell_z \equiv \beta_{yz}\ell_y.$$

12. Conclusion

This paper develops a dynamic stochastic general equilibrium two sector model for a small open economy that can be estimated or calibrated to simulate the macro dynamics of a semi-industrialized developing country like Argentina. We have considered a multilateral non-commodity trade environment, with the U.S.A. and Europe as trade partners and assumed that the Law of One Price does not hold for the goods that the U.S.A. and Europe trade within the model's long run. We show that this makes the bilateral real exchange rate between the U.S.A. and Europe (or, equivalently, the U.S.A.'s multilateral real exchange rate (MRER)) a key fundamental for the SOE's MRER. This fact is especially relevant when the SOE with diversified trade pegs its exchange rate to a single currency, as the recent Argentine experience made painfully evident. The SOE produces and consumes exportables and non tradables. The export sector is perfectly competitive, operating under perfectly flexible export and import prices with instantaneous exchange rate pass-through, but there is monopolistic competition with sticky prices (wages) for non-tradable firms (households). A fraction of these firms (and all households) set prices (wages) optimally, subject to a price/wage adjustment cost function. There is also a subset of non tradable firms that make "rule of thumb" pricing decisions through a basic indexation of their prices to the general non tradable inflation rate and an additional "catching up" component with the price of optimizing firms. This generates a "hybrid" Phillips equation for non tradable inflation which, when the relative price between forward and backward looking non-tradable firms is eliminated, has less constrained coefficients than usual "hybrid" Phillips equations. We considered alternative monetary or foreign exchange policy rules:

two fixed exchange rate regimes, in which the Central Bank fixes the exchange rate either to a single currency (the dollar) or, alternatively, to a trade weighted basket of currencies, and two Inflation Targeting regimes, one with a Pure Float and another with a Managed Float, in which there is a feedback rule for foreign exchange market interventions that reflects a "leaning against the wind" policy. The Inflation Targeting with Managed Float model is also extended in an ad hoc way to include a risk premium on Central Bank domestic currency bonds. This modifies a few of the equations but enables the resulting model to reflect standard accounts of sterilized intervention in the foreign exchange market.

Different venues for future research on DSGE modeling for less developed economies are open. First, the calibration or estimation of this or similar models and the computation of its numerical solutions should be addressed so as to make it possible to evaluate how closely the model can track the real economy. Second, several features of the real economy that have been omitted in this model would make it more realistic and possibly more useful for policy evaluation. In particular, exogenous economic growth could easily be incorporated by suitably modeling the productivity shocks and/or including population growth. Also, foreign trade could be modeled so as to include realistic features such as a slow pass-through and/or local currency pricing. Finally, a financial system could be included to help obtain a better representation of the diverse monetary policy transmission mechanisms. We hope to address some of these issues in future research.

Appendix: Optimizing firms' first order condition and derivation of the forward looking Phillips equation

Inserting the production function (7) and the demand function (14) in the non-tradable period profit function (17) yields:

$$\begin{aligned} \Pi_{i,t}^N = & (P_{N,i,t})^{1-\nu} (P_{N,t})^\nu y_{N,t} \left\{ 1 - x \left(\log \left(\frac{P_{N,i,t}}{P_{N,i,t-1}} \right) \right) \right\} \\ & - W_t F_N^{-1} \left(\frac{(P_{N,i,t})^{-\nu} (P_{N,t})^\nu y_{N,t}}{z_t^N} \right). \end{aligned}$$

Hence, maximizing

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \Pi_{i,t+j}^N,$$

with respect to $P_{N,i,t}$ gives the first order condition:

$$E_t \left\{ \frac{\partial \Pi_{i,t}^N}{\partial P_{N,i,t}} + \Lambda_{t,t+1} \frac{\partial \Pi_{i,t+1}^N}{\partial P_{N,i,t}} \right\} = 0. \quad (195)$$

The two partial derivatives in this expression are:

$$\frac{\partial \Pi_{i,t}^N}{\partial P_{N,i,t}} = y_{N,i,t} \left\{ (1-\nu) [1 - x(\log \pi_{N,i,t})] - x'(\log \pi_{N,i,t}) + \nu \frac{W_t}{P_{N,i,t}} \frac{1}{F_N'(F_N^{-1}(y_{N,t}/z_t^N))} \right\}$$

$$\frac{\partial \Pi_{i,t+1}^N}{\partial P_{N,i,t}} = y_{N,i,t+1} \frac{P_{N,i,t+1}}{P_{N,i,t}} x'(\log \pi_{N,i,t+1}).$$

Inserting these two expressions in (195), eliminating the firm index i (since all optimizing firms are identical) and rearranging, yields:

$$\begin{aligned} 1 - x(\log \pi_{N,t}) + \frac{1}{\nu - 1} \left\{ x'(\log \pi_{N,t}) - E_t \left[\Lambda_{t,t+1} \frac{y_{N,t+1}}{y_{N,t}} \pi_{N,t+1} x'(\log \pi_{N,t+1}) \right] \right\} \\ = \frac{\nu}{\nu - 1} \frac{w_t}{F_N'(F_N^{-1}(y_{N,t}/z_t^N))}. \end{aligned} \quad (196)$$

We log-linearize the left side and right hand side of this equation separately. The l.h.s., which we denominated G_t^P in the text, can be written as a function (which we here denominate $H(\cdot)$) of the logs of $\pi_{N,t}$, $\pi_{N,t+1}$, $\Lambda_{t,t+1}$, and the rate of growth of $y_{N,t+1}$ (which we denominate γ_{t+1}^{yN}):

$$\begin{aligned} H(\log \pi_{N,t}, \log \pi_{N,t+1}, \log \Lambda_{t,t+1}, \log \gamma_{t+1}^{yN}) \equiv 1 - x(\log \pi_{N,t}) \\ + \frac{1}{\nu - 1} \left\{ x'(\log \pi_{N,t}) - E_t \left[e^{\log \Lambda_{t,t+1}} e^{\log \gamma_{t+1}^{yN}} e^{\log \pi_{N,t+1}} x'(\log \pi_{N,t+1}) \right] \right\}. \end{aligned}$$

The linear approximation to $H(\cdot)$ around the steady state is:

$$\bar{H} + \bar{H}_1 \hat{\pi}_{N,t} + \bar{H}_2 E_t \hat{\pi}_{N,t+1} + \bar{H}_3 E_t \hat{\Lambda}_{t,t+1} + \bar{H}_4 E_t \hat{\gamma}_{t+1}^{yN},$$

where \bar{H}_i is the steady state value of the partial derivative of H with respect to its i th variable. Using the properties of the price adjustment cost function (13) and the fact that $\bar{\Lambda} = \beta/\bar{\pi}_N = \beta/\bar{\pi}$, it is straightforward to verify that $\bar{H} = 1$, $\bar{H}_1 = a_F/(\nu - 1) \equiv 1/\gamma_F$, $\bar{H}_2 = -\beta/\gamma_F$, and $\bar{H}_3 = \bar{H}_4 = 0$. Hence

$$G_t^P \approx 1 + \frac{1}{\gamma_F} \hat{\pi}_{N,t} - \frac{\beta}{\gamma_F} E_t \hat{\pi}_{N,t+1}. \quad (197)$$

On the other hand, express the r.h.s. of (196) as a function $K(\cdot)$ of the logs of w_t and $d_t \equiv y_{N,t}/z_t^N$:

$$K(\log w_t, \log d_t) = \mu_F \frac{e^{\log w_t}}{F_N'(F_N^{-1}(e^{\log d_t}))}$$

Then the linear approximation to $K(\cdot)$ is $\bar{K} + \bar{K}_1 \hat{w}_t + \bar{K}_2 \hat{d}_t$. One can verify that $\bar{K} = \bar{K}_1 = 1$ (using (21)) and that $\bar{K}_2 = a_y$ (defined after (22)). Hence,

$$K(\cdot) \approx 1 + \hat{w}_t + a_y \hat{d}_t = 1 + \hat{w}_t + a_y (\hat{y}_{N,t} - \hat{z}_t^N). \quad (198)$$

Equalizing the log-linear approximations to $H(\cdot)$ and $K(\cdot)$ yields the forward looking Phillips equation (22).

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